United Kingdom
Mathematics Trust

Intermediate Mathematical Challenge

Follow-up Competitions

2003 – 2019 Collection

August 18, 2020

Comments and suggestions to 89272376@QQ.com .

IMC Follow-up 1 GreyKang 1
GK Answers 2
Grey Kangaroo 2019 3
Grey Kangaroo 2018 6
Grey Kangaroo 2017 9
Grey Kangaroo 2016 12
Grey Kangaroo 2015 15
Grey Kangaroo 2014 18
Grey Kangaroo 2013 21
Grey Kangaroo 2012 24
Grey Kangaroo 2011 27
Grey Kangaroo 2010 30
Grey Kangaroo 2009 33
Grey Kangaroo 2008 36
Grey Kangaroo 2007 39
Grey Kangaroo 2006 42
Grey Kangaroo 2005 45
Grey Kangaroo 2004 48
Grey Kangaroo 2003 51

IMC Follow-up 2 PinkKang 55
PK Answers 56
Pink Kangaroo 2019 57
Pink Kangaroo 2018 60
Pink Kangaroo 2017 63
Pink Kangaroo 2016 66
Pink Kangaroo 2015 69
Pink Kangaroo 2014 72
Pink Kangaroo 2013 75
Pink Kangaroo 2012 78
Pink Kangaroo 2011 81
Pink Kangaroo 2010 84
Pink Kangaroo 2009 87
Pink Kangaroo 2008 90
Pink Kangaroo 2007 93
Pink Kangaroo 2006 96
Pink Kangaroo 2005 99
Pink Kangaroo 2004 102
Pink Kangaroo 2003 105

IMC Follow-up 3 Y9 Cayley 109
Cayley 2019 111
Cayley 2018 112
Cayley 2017 113
Cayley 2016 114
Cayley 2015 115
Cayley 2014 116
Cayley 2013 117
Cayley 2012 118
Cayley 2011 119
Cayley 2010 120
Cayley 2009 121
Cayley 2008 122
Cayley 2007 123
Cayley 2006 124
Cayley 2005 125
Cayley 2004 126
Cayley 2003 127

IMC Follow-up 4 Y10 Hamilton 131
Hamilton 2019 133
Hamilton 2018 134
Hamilton 2017 135
Hamilton 2016 136
Hamilton 2015 137
Hamilton 2014 138
Hamilton 2013 139
Hamilton 2012 140
Hamilton 2011 141
Hamilton 2010 142
Hamilton 2009 143
Hamilton 2008 144
Hamilton 2007 145
Hamilton 2006 146
Hamilton 2005 147
Hamilton 2004 148
Hamilton 2003 149

IMC Follow-up 5 Y11 Maclaurin 153
Maclaurin 2019 155
Maclaurin 2018 156
Maclaurin 2017 157
Maclaurin 2016 158
Maclaurin 2015 159
Maclaurin 2014 160
Maclaurin 2013 161
Maclaurin 2012 162
Maclaurin 2011 163
Maclaurin 2010 164
Maclaurin 2009 165
Maclaurin 2008 166
Maclaurin 2007 167
Maclaurin 2006 168
Maclaurin 2005 169
Maclaurin 2004 170
Maclaurin 2003 171

IMC Follow-up 4

Year 10 Olympiad - Hamilton

These problems are meant to be challenging! The earlier questions tend to be easier; later questions tend to be more demanding.

Do not hurry, but spend time working carefully on one question before attempting another.

Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **2 hours**.
3. The use of blank or lined paper for rough working, rulers and compasses is allowed; **squared paper, calculators and protractors are forbidden**.
4. You should write your solutions neatly on A4 paper. Staple your sheets together in the top left corner with the Cover Sheet on top and the questions in order.
5. Start each question on a fresh A4 sheet. **Do not hand in rough work**.
6. Your answers should be fully simplified, and exact. They may contain symbols such as π, fractions, or square roots, if appropriate, but not decimal approximations.
7. You should give full written solutions, including mathematical reasons as to why your method is correct. Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.

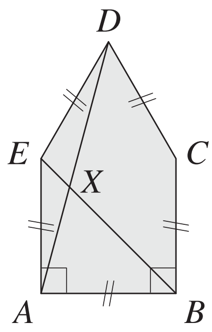
1. A number of couples met and each person shook hands with everyone else present, but not with themselves or their partners.

There were 31 000 handshakes altogether.

How many couples were there?

2. The diagram shows a pentagon $ABCDE$ in which all sides are equal in length and two adjacent interior angles are $90°$. The point X is the point of intersection of AD and BE.

Prove that $DX = BX$.



3. A $4\,\text{cm} \times 4\,\text{cm}$ square is split into four rectangular regions using two line segments parallel to the sides.

How many ways are there to do this so that each region has an area equal to an integer number of square centimetres?

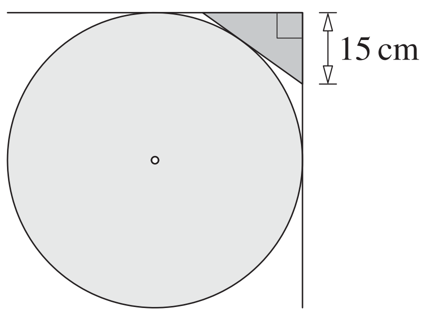
4. Each of A and B is a four-digit palindromic integer, C is a three-digit palindromic integer, and $A - B = C$.

What are the possible values of C?

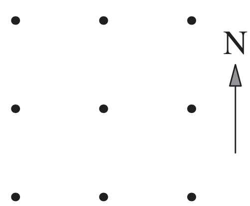
[*A palindromic integer reads the same 'forwards' and 'backwards'.*]

5. The area of the right-angled triangle in the diagram alongside is $60\,\text{cm}^2$. The triangle touches the circle, and one side of the triangle has length 15 cm, as shown.

What is the radius of the circle?



6. Nine dots are arranged in the 2×2 square grid shown. The arrow points north.



Harry and Victoria take it in turns to draw a unit line segment to join two dots in the grid.

Harry is only allowed to draw an east-west line segment, and Victoria is only allowed to draw a north-south line segment. Harry goes first.

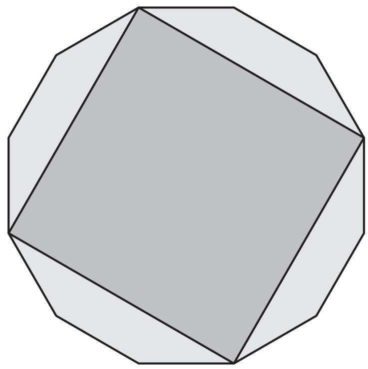
A point is scored when a player draws a line segment that completes a 1×1 square on the grid.

Can either player force a win, no matter how the other person plays?

H1. The positive integers m and n satisfy the equation $20m + 18n = 2018$. How many possible values of m are there?

H2. How many nine-digit integers of the form '*pqrpqrpqr*' are multiples of 24? (*Note that p, q and r need not be different.*)

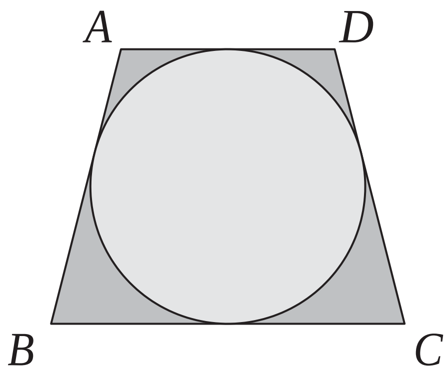
H3. The diagram shows a regular dodecagon and a square, whose vertices are also vertices of the dodecagon.



What is the value of the ratio

area of the square : area of the dodecagon?

H4. The diagram shows a circle and a trapezium $ABCD$ in which AD is parallel to BC and $AB = DC$. All four sides of $ABCD$ are tangents of the circle. The circle has radius 4 and the area of $ABCD$ is 72.

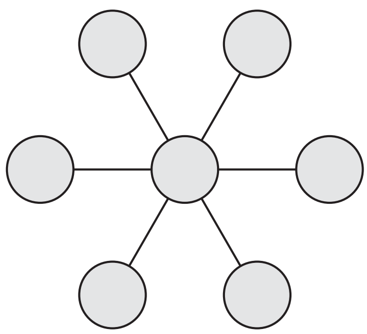


What is the length of AB?

H5. A two-digit number is divided by the sum of its digits. The result is a number between 2.6 and 2.7.

Find all of the possible values of the original two-digit number.

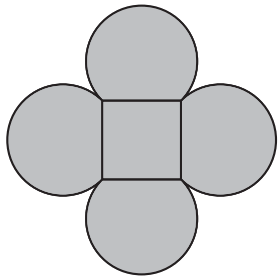
H6. The figure shows seven circles joined by three straight lines.



The numbers 9, 12, 18, 24, 36, 48 and 96 are to be placed into the circles, one in each, so that the product of the three numbers on each of the three lines is the same.

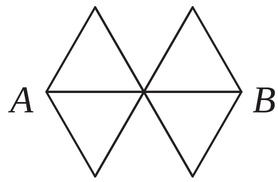
Which of the numbers could go in the centre?

H1. The diagram shows four equal arcs placed on the sides of a square. Each arc is a major arc of a circle with radius 1 cm, and each side of the square has length $\sqrt{2}$ cm.



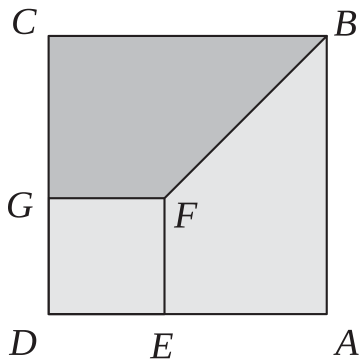
What is the area of the shaded region?

H2. A ladybird walks from A to B along the edges of the network shown. She never walks along the same edge twice. However, she may pass through the same point more than once, though she stops the first time she reaches B.



How many different routes can she take?

H3. The diagram shows squares $ABCD$ and $EFGD$. The length of BF is 10 cm. The area of trapezium $BCGF$ is 35 cm^2.



What is the length of AB?

H4. The largest of four different real numbers is d. When the numbers are summed in pairs, the four largest sums are 9, 10, 12 and 13.

What are the possible values of d?

H5. In the trapezium $ABCD$, the lines AB and DC are parallel, $BC = AD$, $DC = 2AD$ and $AB = 3AD$.

The angle bisectors of $\angle DAB$ and $\angle CBA$ intersect at the point E.

What fraction of the area of the trapezium $ABCD$ is the area of the triangle ABE?

H6. Solve the pair of simultaneous equations

$$x^2 + 3y = 10 \qquad \text{and}$$

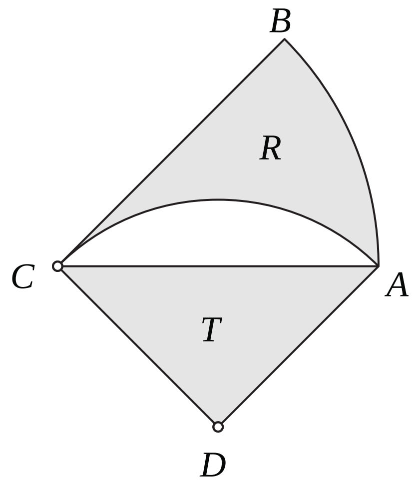
$$3 + y = \frac{10}{x}.$$

H1. No digit of the positive integer N is prime. However, all the single-digit primes divide N exactly.

What is the smallest such integer N?

H2. The diagram shows two arcs. Arc AB is one eighth of a circle with centre C, and arc AC is one quarter of a circle with centre D. The points A and B are joined by straight lines to C, and A and C are joined by straight lines to D.

Prove that the area of the shaded triangle T is equal to the area of the shaded region R.



H3. Alex is given £1 by his grandfather and decides:

(i) to spend at least one third of the £1 on toffees at 5p each;
(ii) to spend at least one quarter of the £1 on packs of bubblegum at 3p each; and
(iii) to spend at least one tenth of the £1 on jellybeans at 2p each.

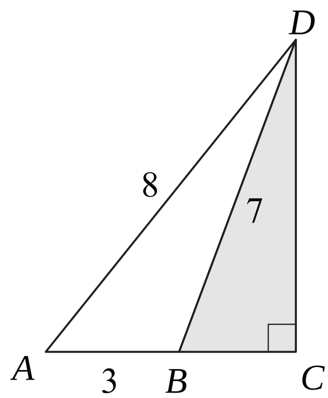
He only decides how to spend the rest of the money when he gets to the shop, but he spends all of the £1 on toffees, packs of bubblegum and jellybeans.

What are the possibilities for the number of jellybeans that he buys?

H4. The diagram shows a right-angled triangle ACD with a point B on the side AC.

The sides of triangle ABD have lengths 3, 7 and 8, as shown.

What is the area of triangle BCD?



H5. James chooses five different positive integers, each at most eight, so that their mean is equal to their median.

In how many different ways can he do this?

H6. Tony multiplies together at least two consecutive positive integers. He obtains the six-digit number N. The left-hand digits of N are '47', and the right-hand digits of N are '74'.

What integers does Tony multiply together?

1. The five-digit integer '$a679b$' is a multiple of 72.

What are the values of a and b?

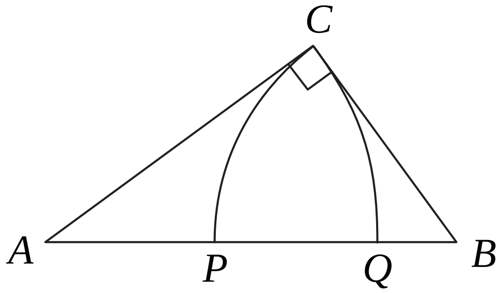
2. In Vegetable Village it costs 75 pence to buy 2 potatoes, 3 carrots and 2 leeks at the Outdoor Market, whereas it costs 88 pence to buy 2 potatoes, 3 carrots and 2 leeks at the Indoor Market.

To buy a potato, a carrot and a leek costs 6 pence more at the Indoor Market than it does at the Outdoor Market.

What is the difference, in pence, between the cost of buying a carrot at the Outdoor Market and the cost of buying a carrot at the Indoor Market?

3. The diagram shows two circular arcs CP and CQ in a right-angled triangle ABC, where $\angle BCA = 90°$. The centres of the arcs are A and B.

Prove that $\frac{1}{2}PQ^2 = AP \times BQ$.



4. The points A, B and C are the centres of three faces of a cuboid that meet at a vertex. The lengths of the sides of the triangle ABC are 4, 5 and 6.

What is the volume of the cuboid?

5. Some boys and girls are standing in a row, in some order, about to be photographed. All of them are facing the photographer. Each girl counts the number of boys to her left, and each boy counts the number of girls to his right.

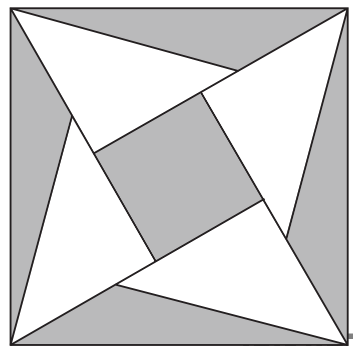
Let the sum of the numbers counted by the girls be G, and the sum of the numbers counted by the boys be B.

Prove that $G = B$.

6. The diagram shows four identical white triangles symmetrically placed on a grey square. Each triangle is isosceles and right-angled.

The total area of the visible grey regions is equal to the total area of the white triangles.

What is the smallest angle in each of the (identical) grey triangles in the diagram?



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