Senior Kangaroo Past Paper

2011 – 2024 Collection

Last updated: September 26, 2025

**United Kingdom Mathematics Trust**

## INSTRUCTIONS

1. Do not open the paper until the invigilator tells you to do so.

2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.

3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.

4. **Use a B or an HB non-propelling pencil** to record your answer to each problem as a three-digit number from 000 to 999.
   Pay close attention to the example on the Answer Sheet that shows how to code your answers.

5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.

6. **Scoring rules:**
   5 marks are awarded for each correct answer;
   There is no penalty for giving an incorrect answer.

7. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

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**Answers:**

|    | 11  | 12  | 13  | 14  | 15  | 16  | 17  | 18  | 19  | 20  | 21  |    |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 1  | 071 | 001 | 009 | 005 | 180 | 121 | 132 | 240 | 403 | 399 | 732 | 1  |
| 2  | 390 | 206 | 110 | 001 | 028 | 950 | 231 | 110 | 256 | 641 | 121 | 2  |
| 3  | 016 | 016 | 025 | 011 | 011 | 027 | 221 | 135 | 210 | 006 | 003 | 3  |
| 4  | 100 | 002 | 012 | 108 | 007 | 125 | 216 | 150 | 087 | 400 | 036 | 4  |
| 5  | 036 | 009 | 025 | 015 | 050 | 009 | 625 | 343 | 130 | 512 | 337 | 5  |
| 6  | 084 | 004 | 002 | 343 | 007 | 160 | 373 | 150 | 338 | 300 | 010 | 6  |
| 7  | 030 | 002 | 013 | 220 | 156 | 315 | 300 | 216 | 012 | 576 | 103 | 7  |
| 8  | 013 | 962 | 007 | 011 | 372 | 007 | 276 | 180 | 004 | 101 | 128 | 8  |
| 9  | 005 | 005 | 060 | 153 | 028 | 256 | 013 | 672 | 100 | 210 | 384 | 9  |
| 10 | 004 | 015 | 003 | 035 | 288 | 103 | 004 | 100 | 140 | 102 | 124 | 10 |
| 11 | 060 | 099 | 128 | 048 | 052 | 183 | 714 | 270 | 094 | 384 | 181 | 11 |
| 12 | 040 | 004 | 011 | 123 | 035 | 144 | 121 | 128 | 105 | 100 | 028 | 12 |
| 13 | 065 | 400 | 008 | 008 | 013 | 143 | 002 | 112 | 198 | 009 | 625 | 13 |
| 14 | 010 | 010 | 992 | 004 | 030 | 035 | 027 | 320 | 216 | 670 | 400 | 14 |
| 15 | 891 | 186 | 008 | 921 | 014 | 110 | 829 | 038 | 129 | 004 | 204 | 15 |
| 16 | 021 | 343 | 030 | 225 | 064 | 087 | 074 | 227 | 317 | 400 | 028 | 16 |
| 17 | 021 | 223 | 150 | 012 | 016 | 649 | 447 | 006 | 246 | 004 | 354 | 17 |
| 18 | 004 | 096 | 804 | 015 | 002 | 441 | 603 | 493 | 014 | 216 | 927 | 18 |
| 19 | 003 | 116 | 741 | 016 | 100 | 421 | 343 | 122 | 735 | 715 | 134 | 19 |
| 20 | 017 | 341 | 050 | 024 | 247 | 116 | 077 | 147 | 295 | 961 | 397 | 20 |
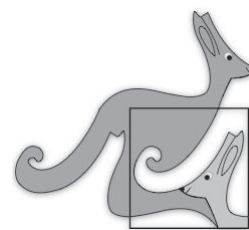
# UK Maths Trust

## Andrew Jobbings Senior Kangaroo

**Wednesday 20 November 2024**

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7. **Your Answer Sheet will be read by a machine.** Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, doodle, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way, or reject the answer sheet.

8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Andrew Jobbings Senior Kangaroo should be sent to:

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ANDREW JOBBINGS SENIOR KANGAROO          Wednesday 20 November 2024

1. The seventh day of an unknown month in an unknown year is a Tuesday. The $n$th day of the following month is the first Tuesday of that month. What is the product of all the possible values of $n$?

2. Each of the following expressions has a different value. Which value is greatest?

$$2 - 0 \times 2 + 4 + 11 \times 20$$
$$2 + 0 \times 2 \times 4 + 11 \times 20$$
$$2 \times 0 + 2 \times 4 + 11 \times 20$$
$$2 \times (0 + 2 + 4) + 11 \times 20$$

3. Kevin begins with eleven stones and a mallet. Every time he hits a stone with the mallet it breaks into exactly eight pieces. Kevin keeps hitting stones with his mallet, but stops before he has 1000 stones. What is the largest number of stones he could end up with?

4. The number 35 can be written as the sum of four distinct primes in two different ways:
$35 = 2 + 3 + 13 + 17$ and $35 = 2 + 3 + 11 + 19$.
What is the *smallest* number that can be written as the sum of four distinct primes in two different ways?

5. What is the value of $|\sqrt{123} - 456| + |\sqrt{123} + 456|$?
*The modulus function, $|x|$ is given by $|x| = x$ for $x \geq 0$, and $|x| = -x$ for $x < 0$.*

6. The equation $x^2 - 70x = 2024$ has solutions $a$, $b$ with $b > a$. What is the value of $10b - a^2$?

7. The triangle $ABC$ is such that $AB = 7$, $BC = 3$, and $CA = 5$.
Furthermore, we are given that $\angle ACB = 120°$.
The area of triangle $ABC$ is $M$. What is the value of $M^2$, correct to the nearest integer?

8. At a football match between Glasgow Belles and Doncaster City, five supporters each made a prediction:
    1. The game will end in a draw;
    2. Glasgow will score;
    3. Glasgow will win;
    4. Glasgow will not lose;
    5. Exactly five goals will be scored.

   What is the maximum number of these predictions that could simultaneously come true?

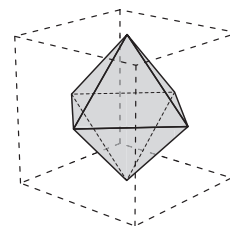9. The number $N = 24^{2024} + 24^{2025}$ has some factors between 1 and 25.
What is the sum of all those integers between 1 and 25 which are *not* factors of $N$?

10. An octahedron is inscribed in a cube of side-length 12. The vertices of the octahedron are at the centre of the faces of the cube. The surface area of the octahedron is $a \times \sqrt{b}$, where $b$ is a prime number. What is the value of $a + 10b$?

11. The number $L = \frac{24!}{20!}$ is one less than a square, $S^2$.
    What is the value of $S$?

12. A solid right circular cone whose base has radius $r$ metres has total surface area (including the circular base) of $A$ metres squared and a volume of $V$ metres cubed.
    The perpendicular height of the cone is $r\sqrt{8}$ metres.
    Given that $\frac{V}{A} = \sqrt{5000}$, what is the value of $r$?
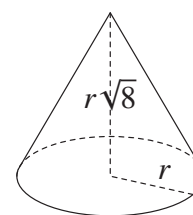
13. Wil goes on a run. Her pedometer is unfortunately rather basic, and records a distance of 40 km, to the nearest integer, and an average speed of 10 km/h, to the nearest integer. Wil works out her greatest possible and least possible times, and expresses the difference of these times as a fraction in its lowest terms, $\frac{a}{b}$.
    What is the value of $a + 2b$?

14. The function $f$ has the property that $f(x + y) = f(x) \times f(y)$ for all values of $x$ and $y$.
    Given that $f(x) > 0$ for all values of $x$, and also that $f(24) = 4096$, what is $f(18)$?

15. Four friends named Ada, Brin, Chris and Debbie are each carrying a different amount of money. These amounts are £111, £222, £333 and £444.

    Ada says:  I am neither carrying the most nor the least.

    Brin says:  I am not carrying the least.

    Chris says:  I am carrying the most.

    Debbie says:  I am carrying the least.

    Exactly one of the friends is lying. How many letters does the lying friend have in her/his name?

16. The prism in the picture is formed of two triangles and three squares. The six vertices are assigned labels 100, 200, 300, 400, 500, and 600 in such a way that the sum of the four labels of the vertices of each square is the same for all three squares. Labels 100 and 500 are shown.
    What number is at the vertex labelled $X$?
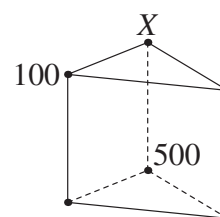
**17.** The parabola $y = x^2 - 10x + 24$ has two distinct $x$-intercepts and a $y$-intercept.
The circle which goes through these three intercepts meets the parabola at a fourth point, $P$.
What is the distance of $P$ from the origin?

**18.** Zarah has a bag containing exactly four blue counters and exactly four red counters, and a large supply, outside the bag, of extra blue and red counters.
She may, on any given *move*, do one of four different things:

- Remove a blue counter from the bag;
- Remove a red counter from the bag;
- Add a blue counter to the bag;
- Add a red counter to the bag.

At each move, she selects one of those four options at random and with equal probability. The probability that after four consecutive moves, Zarah's bag again contains exactly four blue counters and exactly four red counters is $\frac{V}{W}$ when written in its lowest terms.
What is the value of $10V + W$?

**19.** A regular hexagon $PQRSTU$ is shown. The midpoint of $UP$ is $M$. Point $I$ is the intersection between line segments $RM$ and $QT$, and point $J$ is the intersection between line segments $SM$ and $QT$. The area of triangle $RMS$ is 920. What is the area of triangle $QMI$?
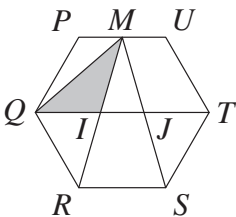


**20.** Each cell in this cross-number can be filled with a non-zero digit so that all of the conditions in the clues are satisfied. The digits used are not necessarily distinct.



ACROSS
1. A prime.
3. The answer to this Kangaroo question.
5. Shares a two-digit common factor with 1 DOWN.

DOWN
1. The sum of two different sixth powers.
2. 101 less than a square number.
4. The sum of all the digits in this crossnumber, once correctly completed.

1. A particular flag is in the shape of a rectangle divided into five smaller congruent rectangles as shown. When written in its lowest terms, the ratio of the side lengths of the smaller rectangle is $\lambda : 1$, where $\lambda < 1$.
   What is the value of $360\lambda$?
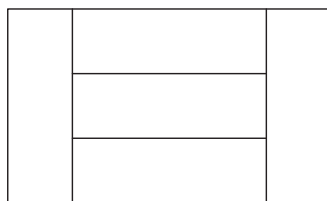
   

2. The integers $m$ and $n$ satisfy the equation $3^m \times n = 7! + 8! + 9!$.
   What is the smallest possible value for $n$?

3. How many of the integers from $2^{10}$ to $2^{18}$ inclusive are divisible by $2^9$?

4. Months of the year are usually labelled numerically by '01' for January, '02' for February, and so on, through to '12' for December. Lydia notices that during January, the number of letters in the name of the month is greater than the month's numerical label (i.e. $7 > 1$). For how many days during 2024 will the date have that property?

5. The number $C$ is defined as the sum of all the positive integers $n$ such that $n - 6$ is the second largest factor of $n$. What is the value of $11C$?

6. Five cards have the numbers 101, 102, 103, 104 and 105 on their fronts.

   $\boxed{101}$    $\boxed{102}$    $\boxed{103}$    $\boxed{104}$    $\boxed{105}$

   On the reverse, each card has a statement printed as follows:
   101: The statement on card 102 is false
   102: Exactly two of these cards have true statements
   103: Four of these cards have false statements
   104: The statement on card 101 is false
   105: The statements on cards 102 and 104 are both false
   What is the total of the numbers shown on the front of the cards with TRUE statements?

7. A container in the shape of a rectangular box is partially filled with $120 \, \text{m}^3$ of water. The depth of the water is either 2 m, 3 m or 5 m, depending on which side of the box is on the ground.
   What is the volume of the container in $\text{m}^3$?

8. The product of all the factors of $5^{15}$ is $5^P$.
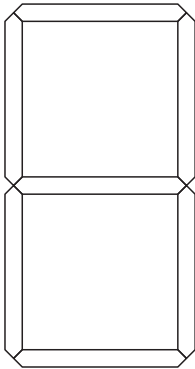   What is the value of $P$?

**9.** What is the lowest prime number that is thirteen more than a cube?

**10.** The smallest four two-digit primes are written in different squares of a $2 \times 2$ table.
The sums of the numbers in each row and column are calculated.
Two of these sums are 24 and 28.
The other two sums are $c$ and $d$, where $c < d$.
What is the value of $5c + 7d$?



**11.** The town of Coppersville uses some special coins. Edward has five $5\,\mathrm{p}$ coins, four $4\,\mathrm{p}$ coins, three $3\,\mathrm{p}$ coins, two $2\,\mathrm{p}$ coins and one $1\,\mathrm{p}$ coin. The smallest amount he cannot pay with his coins alone is $W$ p.
What is the value of $W$?

**12.** How many positive integers are there whose digits do not include 0, and whose digits have sum 6?

**13.** On a square $ABCD$ a line segment $BE$ is drawn such that the point $E$ lies on the side $CD$. The perimeter of triangle $BCE$ is three-quarters of the perimeter of the square $ABCD$. The ratio of lengths $CE : CD$ is $\lambda : 1$.
What is the value of $960 \times \lambda$?

**14.** Three different numbers are chosen at random from the list 1, 3, 5, 7, 9, 11, 13, 15, 17, 19. The probability that one of them is the mean of the other two is $p$.
What is the value of $\frac{120}{p}$?

**15.** Kiran is designing a game which involves a bag of twenty-one marbles. Some of the marbles are blue, the rest are red. To play the game, two marbles are drawn out. The game is won if at least one red marble is drawn. To ensure the probability of the game being won is exactly one-half, Kiran uses $B$ blue marbles and $R$ red marbles.
What is the value of $B^2 + R^2$?

**16.** Stephen's calculator displays only one digit, as shown in the diagram. Unfortunately, the calculator is broken. Each time he switches it on, each of the seven bars will either illuminate (show up) or not, with probability 0.5. The resultant display correctly shows one of the ten digits 0 - 9 with probability $\frac{a}{b}$.

Given that $\frac{a}{b}$ is written in its lowest terms, what is the value of $9a + 2b$?

**17.** The integer part of $\sqrt{72 + \sqrt{72 + \sqrt{72 + \sqrt{72 + \sqrt{72 + \sqrt{72}}}}}}$ is $A$.

What is the value of $50A$?

**18.** The number 2024 may be split into its first two digits and its last two digits to form the numbers 20 and 24. The highest common factor of these numbers, HCF(20,24) is equal to 4. Similarly, 2025 may be split, and HCF(20,25)=5.

For how many remaining years this century (i.e. after 2025 and up to and including 2099) will this highest common factor be equal to one?

**19.** Next year's 'leap day', 29th February 2024, may be written in 'ddmmyy' format as the number 290224. What is the sum of all the two-digit factors of 290224?

**20.** Each cell in this cross-number can be filled with a non-zero digit so that all of the conditions in the clues are satisfied.

The digits used are not necessarily distinct.

ACROSS
1.   Four less than a factor of 105.
3.   One more than a palindrome.
5.   The square-root of the answer to this Kangaroo question.

DOWN
1.   Two less than a square.
2.   Four hundred less than a cube.
4.   Six less than the sum of the answers to two of the other clues.

What is the square of the answer to 5 ACROSS?

**1.** The number $x$ satisfies $x^2 - 4x + 2 = 0$. What is the value of $x + \frac{2}{x}$?

**2.** The sum of the ages of Tom and John is 23, the sum of the ages of John and Alex is 24 and the sum of the ages of Tom and Alex is 25. What is the sum of the squares of their three ages?

**3.** Zia conducted a study of the number of coins each member of her club had in their pocket. To her surprise, nobody had zero coins and exactly seven people had only one coin each. Everybody else had at least two coins. She drew a pie chart of her results, and to her surprise found each sector had an angle which was an integer ten degrees larger than its immediate predecessor. The smallest sector in the pie chart had an angle of $35^o$. How many people are in Zia's club?

**4.** The rectangles $S_1$ and $S_2$ are drawn on a set of coordinate axes. $S_1$ has vertices at $(0, y)$; $(x, y)$; $(x, 4)$; and $(0, 4)$, where $x < 9$ and $y < 4$. $S_2$ has vertices at $(x, 0)$; $(9, 0)$; $(9, y)$; and $(x, y)$. The rectangles are equal in area. What is the value of $\frac{360x}{y}$?

**5.** Paul is painting a wall. He knows the area of the wall is 1920 square metres, correct to the nearest ten. He uses tins of paint, each of which can cover 18 square metres, correct to the nearest integer.
He needs to paint the wall completely, and still have at least half a tin of paint left over for any minor repairs.
What is the smallest number of tins he must buy to be sure of having enough paint?

**6.** A cube is dissected into 6 pyramids by connecting a given point in the interior of the cube with each vertex of the cube, so that each face of the cube forms the base of a pyramid. The volumes of five of these pyramids are 200, 500, 1000, 1100 and 1400. What is the volume of the sixth pyramid?

**7.** The positive integer $N$ has exactly six distinct (positive) factors including 1 and $N$. The product of five of these is 6075. What is the value of the sixth factor?

**8.** In a right-angled triangle $ABC$ (with right angle at $A$) the bisectors of the acute angles intersect at point $P$. The distance from $P$ to the hypotenuse is $\sqrt{80000}$. What is the distance from $P$ to $A$?

**9.** Using the 24-hour clock, the time 16:11:22 on the date 16/11/22 (i.e. the 16th of November 2022) has hours, minutes and seconds equal to date, month and (two-digit) year respectively. Let S be the number of seconds which will elapse after that date and time until this phenomenon next occurs.
What is the value of $\sqrt{S}$?

10. The equations $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have real roots. It is known that the sum of the squares of the roots of the first equation is equal to the sum of the squares of the roots of the second one, and that $a \neq b$. Find the value of $500 + 10(a + b)$.

11. A quadrilateral contains an inscribed circle (i.e. a circle tangent to the four sides of the quadrilateral). The ratio of the perimeter of the quadrilateral to that of the circle is 4 : 3. The ratio of the area of the quadrilateral to that of the circle, expressed in its lowest terms, is $a : b$. What is the value of $100a + 49b$?

12. Five cards have the numbers 101, 102, 103, 104 and 105 on their fronts.

$$\boxed{101} \quad \boxed{102} \quad \boxed{103} \quad \boxed{104} \quad \boxed{105}$$

On the reverse, each card has one of five different positive integers: $a$, $b$, $c$, $d$ and $e$ respectively. We know that $a + 2 = b - 2 = 2c = \frac{d}{2} = e^2$.
Gina picks up the card which has the largest integer on its reverse. What number is on the front of Gina's card?

13. A sequence is given by $x_1 = 2$, $x_{n+1} = \frac{x_n{}^{x_n}}{32^{n-1}}$. What is $\frac{x_4}{128}$?

14. A class consists of pupils who each wear either a red tie or a green tie. There are two more pupils wearing green ties than wearing red ties. Two pupils are chosen at random. The probability of selecting two pupils of different tie-colour is exactly treble the probability of selecting two pupils who both wear red ties. Given that R is the number of pupils wearing a red tie, and that $R > 0$, determine $R^3 + R^2 + R$.

15. The perimeter of the square in the figure is 40. The perimeter of the larger equilateral triangle in the figure is $a + b\sqrt{p}$, where $p$ is a prime number. What is the value of $7a + 5b + 3p$?
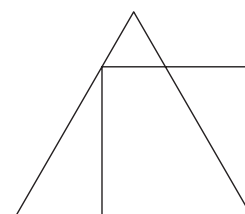
16. On the island of Friends and Foes, every citizen is either a Friend (who always tells the truth) or a Foe (who always lies). Seven citizens are sitting in a circle. Each declares "I am sitting between two Foes". How many Friends are there in the circle?

17. What is the smallest three-digit positive integer which can be written in the form $pq^2r$, where $p$, $q$ and $r$ are distinct primes?

18. The number $\dfrac{20! \times 22!}{16! \times 11!}$ has $N$ prime factors, which are not necessarily distinct.

What is the value of $N(N - 2)$?

**19.** How many different real number solutions are there to the following equation?

$$\left(x^2 - 8x + 15\right)^{\left(x^5 - 5x^3 + 4x\right)} = 1$$

**20.** Each cell in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are satisfied. The digits used are not necessarily distinct.
Determine T, the sum of all six answers to the clues. The answer to this question is $0.5 \times T$.

|   |   |   |
|---|---|---|
| 1 | 2 |   |
| 3 |   | 4 |
|   | 5 |   |

ACROSS:
1. Two less than a prime.
3. Exactly 100 more than its largest proper factor.
5. A multiple of 13.

DOWN:
1: A fourth power.
2: A cube.
4: Not prime, not square, not even.

**1.** Adil was born in 2015. His younger sister Bav was born in 2018. What is the minimum number of days by which Adil is older than Bav?

**2.** The total $T$ is obtained as the sum of the integers from 2006 to 2036 inclusive. What is the sum of all the prime factors of $T$?

**3.** How many of the figures shown can be drawn with one continuous line without drawing a segment twice?
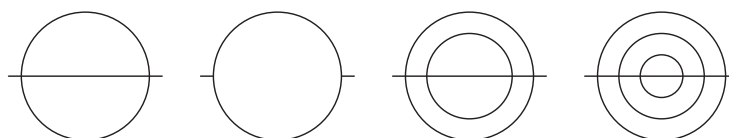


**4.** On each side of a right-angled triangle, a semicircle is drawn with that side as a diameter. The areas of the three semicircles are $x^2$, $3x$ and 180 where $x^2$ and $3x$ are both less than 180. What is the area of the smallest semicircle ?

**5.** $T = \sqrt{(2021 + 2021) + (2021 - 2021) + (2021 \times 2021) + (2021 \div 2021)}$.
What is the largest prime factor of T?

**6.** Into how many regions do the $x$-axis and the graphs of $y = 2 - x^2$ and $y = x^2 - 1$ split the plane?

**7.** Five cards have the numbers 101, 102, 103, 104 and 105 on their fronts.

$$\boxed{101} \quad \boxed{102} \quad \boxed{103} \quad \boxed{104} \quad \boxed{105}$$

On the reverse, each card has one of five different positive integers: $a$, $b$, $c$, $d$ and $e$ respectively.
We know that $c = be$, $a + b = d$ and $e - d = a$.
Frankie picks up the card which has the largest integer on its reverse. What number is on the front of Frankie's card?

**8.** The geometric mean of a set of $n$ positive numbers is defined as the $n$-th root of the product of those numbers.
Yasmeen writes down a set of four numbers which have a geometric mean of 2048.
Zak writes down a set of four numbers which have a geometric mean of 8.
What is the geometric mean of the combined set of the eight numbers written by Yasmeen and Zak?

**9.** In the figure shown there are three concentric circles and two perpendicular diameters. The three shaded regions have equal area. The radius of the small circle is 2. The product of the three radii is $Y$.
What is the value of $Y^2$?
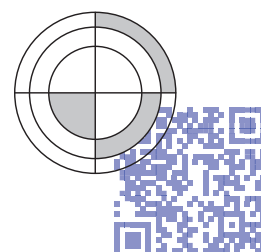
**10.** A dealer bought two cars. He sold the first one for 40% more than he paid for it and the second one for 60% more than he paid for it. The total sum he received for the two cars was 54% more than the total sum he paid for them. When written in its lowest terms, the ratio of the prices the dealer paid for the first and the second car was $a : b$. What is the value of $11a + 13b$?

**11.** Billie has a die with the numbers 1, 2, 3, 4, 5 and 6 on its six faces.
Niles has a die which has the numbers 4, 4, 4, 5, 5 and 5 on its six faces.
When Billie and Niles roll their dice the one with the larger number wins. If the two numbers are equal it is a draw.
The probability that Niles wins, when written as a fraction in its lowest terms, is $\frac{p}{q}$. What is the value of $7p + 11q$?

**12.** There are 2021 balls in a crate. The balls are numbered from 1 to 2021. Erica works out the digit sum for each ball. For example, the digit sum of 2021 is 5, since $2 + 0 + 2 + 1 = 5$. Erica notes that balls with equal digit sums have the same colour and balls with different digit sums have different colours.
How many different colours of balls are there in the crate?

**13.** A multiplication table of the numbers 1 to 10 is shown. What is the sum of all the odd products in the complete table?

| $\times$ | 1 | 2 | 3 | $\cdots$ | 10 |
|---|---|---|---|---|---|
| 1 | 1 | 2 | 3 | $\cdots$ | 10 |
| 2 | 2 | 4 | 6 | $\cdots$ | 20 |
| $\vdots$ | $\vdots$ | | | | $\vdots$ |
| 10 | 10 | 20 | 30 | $\cdots$ | 100 |

**14.** The graph of $(x^2 + y^2 - 2x)^2 = 2(x^2 + y^2)^2$ meets the $x$-axis in $p$ different places and meets the $y$-axis in $q$ different places.
What is the value of $100p + 100q$?

**15.** Which is the lowest numbered statement which is true?
Statement 201: "Statement 203 is true".
Statement 202: "Statement 201 is true".
Statement 203: "Statement 206 is false".
Statement 204: "Statement 202 is false".
Statement 205: "None of the statements 201, 202, 203 or 204 are true".
Statement 206: "1 + 1 = 2".

**16.** A polygon is said to be *friendly* if it is regular and it also has angles that when measured in degrees are either integers or *half-integers* (i.e. have a decimal part of exactly 0.5). How many different friendly polygons are there?
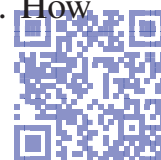
**17.** Find the value of $R$, given that the numbers $Q$ and $R$ are defined as:

$Q = 202^1 + 20^{21} + 2^{021}$;
$R$ is the remainder when $Q$ is divided by 1000.

**18.** Some years, like 2022, have the property that they use at most two different digits. How many years between 1 and 9999, inclusive, have this property?

**19.** The function $f(x)$ is defined as $f(x) = \frac{x-1}{x+1}$.
The equation $f(x^2) \times f(x) = 0.72$ has two solutions $a$ and $b$, where $a > b$.
What is the value of $19a + 7b$?

**20.** Each cell in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are satisfied. The digits used are not necessarily distinct.
What is the answer to 2 DOWN?

| 1 | 2 | |
|---|---|---|
| 3 | | 4 |
| | 5 | |

ACROSS

1. A prime which is the sum of two squares

3. Twice the answer to 2 DOWN

DOWN

1. $p \times q$, where $p$, $q$ are prime and $q = p + 4$

4. 60% of 5 ACROSS

1. What is the difference between the greatest and the least of the following five quantities?

   $20 + 20$       $20 \times 20$       $202 + 0$       $(2^0)^{(2^0)}$       $20 + 2 + 0$

2. The positive integers $x$ and $y$ satisfy the equation $yx^2 + xy^2 = 70$. What is the value of $x^4 + y^4$?

3. How many distinct integer solutions $(x, y)$ are there to the equation $5^1 + 4^2 + 3^3 + 2^4 = x^y$?

4. Two identical cylindrical sheets are cut open along the dotted lines and glued together to form one bigger cylindrical sheet, as shown. The smaller sheets each enclose a volume of 100. What volume is enclosed by the larger sheet?
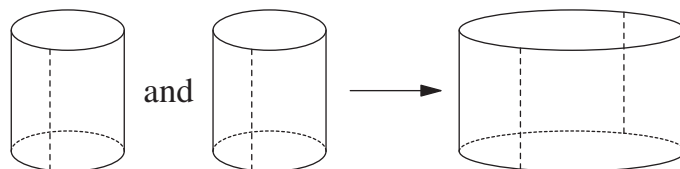
   

5. Let $a^b = \frac{1}{8}$. What is the value of $a^{-3b}$?

6. For what value of $x$ does the expression $x^2 - 600x + 369$ take its minimum value?

7. Margot writes the numbers 1,2,3,4,5,6,7 and 8 in the top row of a table, as shown. In the second row she plans to write the same set of numbers, in any order.
   Each number in the third row is obtained by finding the sum of the two numbers above it.
   In how many different ways can Margot complete row 2 so that every entry in row 3 is even?

   | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
   |---|---|---|---|---|---|---|---|
   |   |   |   |   |   |   |   |   |
   |   |   |   |   |   |   |   |   |

8. The number $(2^{222})^5 \times (5^{555})^2$ is $Q$ digits long. What is the largest prime factor of $Q$?

9. The radii of two concentric circles are in the ratio 1 : 3.
   $AC$ is a diameter of the larger circle. $BC$ is a chord of the larger circle and is tangent to the smaller circle. $AB$ has length 140.
   What is the radius of the larger circle?

10. What is the smallest 3-digit positive integer $N$ such that $2^N + 1$ is a multiple of 5?

11. A circle is drawn inside a regular hexagon so that it touches all six sides of the hexagon.
    The area of the circle is $\pi \times 64\sqrt{3}$. What is the area of the hexagon?

12. What is the value of $\sqrt{20212020 \times 20202021 - 20212021 \times 20202020}$ ?

13. How many ordered triples of positive integers $(x, y, z)$ satisfy $(x^y)^z = 1024$?

**14.** Let $a$, $b$, $c$ and $d$ be distinct positive integers such that $a + b$, $a + c$ and $a + d$ are all odd and are all square. Let $L$ be the least possible value of $a + b + c + d$. What is the value of $10L$?

**15.** On an island, kangaroos are always either grey or red. One day, the number of grey kangaroos increased by 28% while the number of red kangaroos decreased by 28%. The ratios of the two types of kangaroos were exactly reversed. By what percentage did the total number of kangaroos change?

**16.** A square fits snugly between the horizontal line and two touching circles of radius 1000, as shown. The line is tangent to the circles. What is the side-length of the square?

**17.** How many solutions does equation $|||x - 1| - 1| - 1| = 1$ have?
*The modulus function $|x|$ evaluates the absolute value of a number; for example $|6| = |-6| = 6$.*

**18.** The operation $\diamond$ is defined on two positive whole numbers as the number of distinct prime factors of the product of the two numbers. For example $8 \diamond 15 = 3$.
What is the cube of the value of $(720 \diamond 1001)$?

**19.** A random number generator gives outputs of 1, 2, 3, 4 and 5 with equal probability.
The values of $a$, $b$ and $c$ are each chosen by running the generator once.
The probability that $a \times b + c$ is even can be written as a fraction in its lowest terms as $\frac{N}{D}$.
What is the value of $10N + D$?

**20.** Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.
What is the answer to 3 ACROSS?
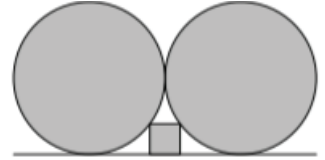
|   1 |   2 |   |
|---|---|---|
|   3 |   |   4 |
|   |   5 |   |

ACROSS
1. A multiple of 7
3. The answer to this Question
5. More than 10

DOWN
1. A multiple of a square of an odd prime; neither a square nor a cube
2. The internal angle of a regular polygon; the exterior angle is between $10^o$ and $20^o$
4. A proper factor of 5 ACROSS but not a proper factor of 1 DOWN
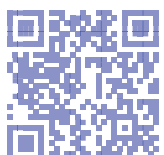
**1.** What is the sum of all the factors of 144?

**2.** When I noticed that $2^4 = 4^2$, I tried to find other pairs of numbers with this property. Trying 2 and 16, I realised that $2^{16}$ is larger than $16^2$. How many times larger is $2^{16}$?

**3.** The two diagonals of a quadrilateral are perpendicular. The lengths of the diagonals are 14 and 30. What is the area of the quadrilateral?

**4.** The integer $n$ satisfies the inequality $n + (n + 1) + (n + 2) + \cdots + (n + 20) > 2019$.

What is the minimum possible value of $n$?

**5.** Identical regular pentagons are arranged in a ring. The partially completed ring is shown in the diagram. Each of the regular pentagons has a perimeter of 65. The regular polygon formed as the inner boundary of the ring has a perimeter of $P$. What is the value of $P$?
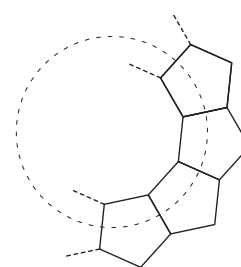
**6.** For natural numbers $a$ and $b$ we are given that $2019 = a^2 - b^2$. It is known that $a < 1000$. What is the value of $a$?

**7.** How many positive? integers $n$ exist such that both $\frac{n+1}{3}$ and $3n + 1$ are three-digit integers?

**8.** The function $J(x)$ is defined by:

$$J(x) = \begin{cases} 4 + x & \text{for } x \leq -2, \\ -x & \text{for } -2 < x \leq 0, \\ x & \text{for } x > 0. \end{cases}$$
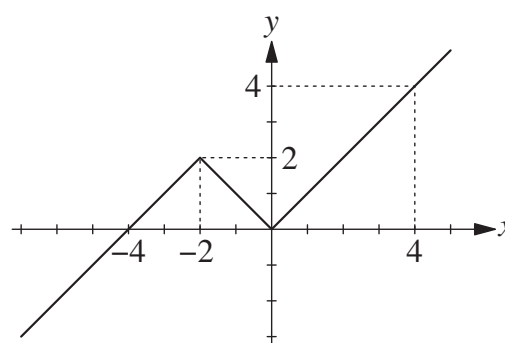
How many distinct real solutions has the equation $J(J(J(x))) = 0$?

**9.** What is the smallest three-digit number $K$ which can be written as $K = a^b + b^a$, where both $a$ and $b$ are one-digit positive integers?

**10.** What is the value of $\sqrt{13 + \sqrt{28 + \sqrt{281}}} \times \sqrt{13 - \sqrt{28 + \sqrt{281}}} \times \sqrt{141 + \sqrt{281}}$ ?

**11.** In the triangle $ABC$ the points $M$ and $N$ lie on the side $AB$ such that $AN = AC$ and $BM = BC$.

We know that $\angle MCN = 43°$.

Find the size in degrees of $\angle ACB$.
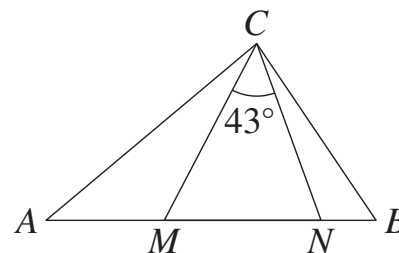


**12.** What is the value of $A^2 + B^3 + C^5$, given that:

$A = \sqrt[3]{16\sqrt{2}}$

$B = \sqrt{9\sqrt[3]{9}}$

$C = \left[\left(\sqrt[5]{2}\right)^2\right]^2$

**13.** The real numbers $a$ and $b$, where $a > b$, are solutions to the equation $3^{2x} - 10 \times 3^{x+1} + 81 = 0$. What is the value of $20a^2 + 18b^2$?

**14.** A number $N$ is the product of three distinct primes. How many distinct factors does $N^5$ have?

**15.** Five Bunchkins sit in a horizontal field. No three of the Bunchkins are sitting in a straight line. Each Bunchkin knows the four distances between her and each of the others. Each Bunchkin calculates and then announces the total of these distances. These totals are 17, 43, 56, 66 and 76. A straight line is painted joining each pair of Bunchkins. What is the total length of paint required?

**16.** The real numbers $x$ and $y$ satisfy the equations:

$$xy - x = 180 \quad \text{and} \quad y + xy = 208.$$

Let the two solutions be $(x_1, y_1)$ and $(x_2, y_2)$.
What is the value of $x_1 + 10y_1 + x_2 + 10y_2$?

**17.** In triangle $ABC$, $\angle BAC$ is $120°$. The length of $AB$ is 123. The point $M$ is the midpoint of side $BC$. The line segments $AB$ and $AM$ are perpendicular.
What is the length of side $AC$?

**18.** An integer is said to be *chunky* if it consists only of non-zero digits by which it is divisible when written in base 10.
For example, the number 936 is Chunky since it is divisible by 9, 3 and 6.
How many chunky integers are there between 13 and 113?

**19.** The square $ABCD$ has sides of length 105. The point $M$ is the midpoint of side $BC$. The point $N$ is the midpoint of $BM$. The lines $BD$ and $AM$ meet at the point $P$. The lines $BD$ and $AN$ meet at the point $Q$.
What is the area of triangle $APQ$?

**20.** Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.
What is the answer to 3 ACROSS?



ACROSS
1. A composite factor of 1001
3. Not a palindrome
5. $pq^3$ where $p, q$ prime and $p \neq q$

DOWN
1. One more than a prime, one less than a prime
2. A multiple of 9
4. $p^3q$ using the same $p, q$ as 5 ACROSS

**1.** My age is a two-digit number that is a power of 5. My cousin's age is a two-digit number that is a power of 2. The sum of the digits of our ages is an odd number.

What is the product of the digits of our ages?

**2.** Let $K$ be the largest integer for which $n^{200} < 5^{300}$. What is the value of $10K$?

**3.** In triangle $ABC$, we are given that $AC = 5\sqrt{2}$, $BC = 5$ and $\angle BAC = 30°$.

What is the largest possible size in degrees of $\angle ABC$?

**4.** In a list of five numbers, the first number is 60 and the last number is 300. The product of the first three numbers is 810 000, the product of the three in the middle is 2 430 000 and the product of the last three numbers is 8 100 000.

Which number is third in the list?

**5.** Rachel and Steven play games of chess. If either wins two consecutive games s/he is declared the champion.

The probability that Rachel will win any given game is 0.6.

The probability that Steven will win any given game is 0.3.

There is a 0.1 probability that any given game is drawn.

The probability that neither is the champion after at most three games is $P$. Find the value of $1000P$.

**6.** The line segments $PQRS$ and $WXYS$ intersect circle $C_1$ at points $P$, $Q$, $W$ and $X$.
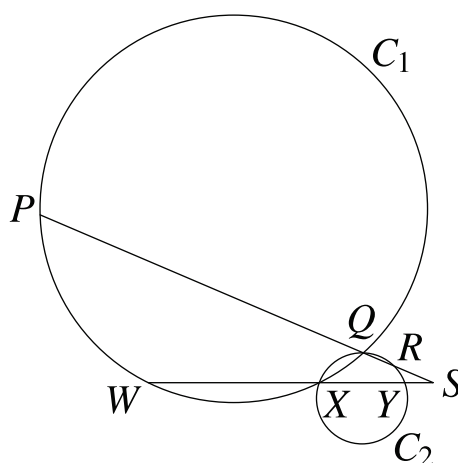


The line segments intersect circle $C_2$ at points $Q$, $R$, $X$ and $Y$. The lengths $QR$, $RS$ and $XY$ are 7, 9 and 18 respectively. The length $WX$ is six times the length $YS$.

What is the sum of the lengths of $PS$ and $WS$?

**7.** The volume of a cube in cubic metres and its surface area in square metres is numerically equal to four-thirds of the sum of the lengths of its edges in metres.

What is the total volume in cubic metres of twenty-seven such cubes?

8. An integer $x$ satisfies the inequality $x^2 \leq 729 \leq -x^3$. $P$ and $Q$ are possible values of $x$. What is the maximum possible value of $10(P - Q)$?

9. The two science classes 7A and 7B each consist of a number of boys and a number of girls. Each class has exactly 30 students.

   The girls in 7A have a mean score of 48. The overall mean across both classes is 60.

   The mean score across all the girls of both classes is also 60.

   The 5 girls in 7B have a mean score that is double that of the 15 boys in 7A.

   The mean score of the boys in 7B is $\mu$. What is the value of $10\mu$?

10. The function SPF($n$) denotes the sum of the prime factors of $n$, where the prime factors are not necessarily distinct. For example, $120 = 2^3 \times 3 \times 5$, so SPF($120$) $= 2 + 2 + 2 + 3 + 5 = 14$.

    Find the value of SPF($2^{22} - 4$).

11. A sequence $U_1, U_2, U_3, \ldots$ is defined as follows:
    - $U_1 = 2$;
    - if $U_n$ is prime then $U_{n+1}$ is the smallest positive integer not yet in the sequence;
    - if $U_n$ is not prime then $U_{n+1}$ is the smallest prime not yet in the sequence.

    The integer $k$ is the smallest such that $U_{k+1} - U_k > 10$.

    What is the value of $k \times U_k$?

12. The diagram shows a 16 metre by 16 metre wall. Three grey squares are painted on the wall as shown.
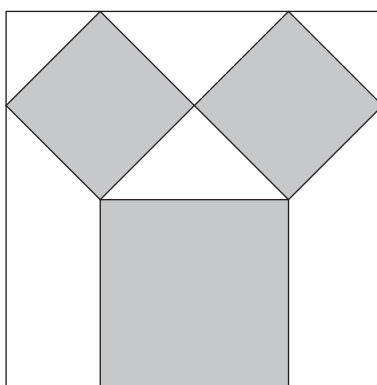


   The two smaller grey squares are equal in size and each makes an angle of $45°$ with the edge of the wall. The grey squares cover a total area of $B$ metres squared.

   What is the value of $B$?

13. A nine-digit number is odd. The sum of its digits is 10. The product of the digits of the number is non-zero. The number is divisible by seven.

    When rounded to three significant figures, how many millions is the number equal to?

**14.** A square $ABCD$ has side 40 units. Point $F$ is the midpoint of side $AD$. Point $G$ lies on $CF$ such that $3CG = 2GF$.

What is the area of triangle $BCG$?

**15.** In the sequence $20, 18, 2, 20, -18, \ldots$ the first two terms $a_1$ and $a_2$ are 20 and 18 respectively. The third term is found by subtracting the second from the first, $a_3 = a_1 - a_2$. The fourth is the sum of the two preceding elements, $a_4 = a_2 + a_3$. Then $a_5 = a_3 - a_4$, $a_6 = a_4 + a_5$, and so on.

What is the sum of the first 2018 terms of this sequence?

**16.** A right-angled triangle has sides of integer length. One of its sides has length 20. Toni writes down a list of all the different possible hypotenuses of such triangles.

What is the sum of all the numbers in Toni's list?

**17.** Sarah chooses two numbers $a$ and $b$ from the set $\{1, 2, 3, \ldots, 26\}$. The product $ab$ is equal to the sum of the remaining 24 numbers.

What is the difference between $a$ and $b$?

**18.** How many zeros are there at the end of $\dfrac{2018!}{30! \times 11!}$?

**19.** Shan solves the simultaneous equations

$$xy = 15 \text{ and } (2x - y)^4 = 1$$

where $x$ and $y$ are real numbers. She calculates $z$, the sum of the squares of all the $y$-values in her solutions.

What is the value of $z$?

**20.** Determine the value of the integer $y$ given that $y = 3x^2$ and

$$\frac{2x}{5} = \cfrac{1}{1 - \cfrac{2}{3 + \cfrac{1}{4 - \cfrac{5}{6 - x}}}}$$

1. An integer is to be written in each circle of the network shown. The integers must be written so that the sum of the numbers at the end of each line segment is the same. Two of the integers have already been written. What is the total of all the integers in the completed diagram?
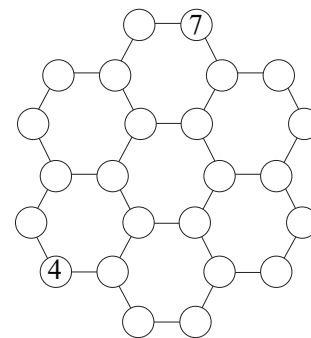
2. Three sportsmen called Primus, Secundus and Tertius take part in a race every day. Primus wears the number '1' on his shirt, Secundus wears '2' and Tertius wears '3'.

    On Saturday Primus wins, Secundus is second and Tertius is third. Using their shirt numbers this result is recorded as '123'.

    On Sunday Primus starts the race in the lead with Secundus in second. During Sunday's race Primus and Secundus change places exactly 9 times, Secundus and Tertius change places exactly 10 times while Primus and Tertius change places exactly 11 times.

    How will Sunday's result be recorded?

3. All three-digit positive integers whose digit sum is 5 are listed in ascending order. What is the median of this list?

4. The figure shows a shape consisting of a regular hexagon of side 18 cm, six triangles and six squares. The outer perimeter of the shape is $P$ cm. What is the value of $P$?
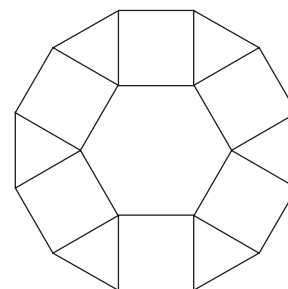
5. The figure shows a quadrilateral $ABCD$ in which $AD = DC$ and $\angle ADC = \angle ABC = 90°$. The point $E$ is the foot of the perpendicular from $D$ to $AB$. The length $DE$ is 25. What is the area of quadrilateral $ABCD$?
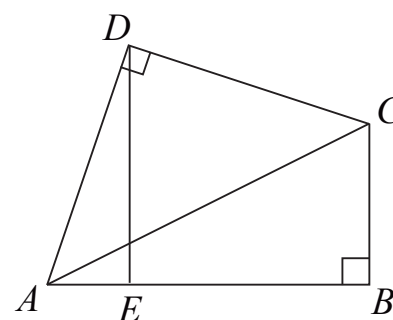
6. Winnie wrote all the integers from 1 to 2017 inclusive on a board. She then erased all the integers that are a multiple of 3. Next she reinstated all those integers that are a multiple of 6. Finally she erased all integers then on the board which are a multiple of 27. Of the 2017 integers that began in the list, how many are now missing?

7. Three rectangles are placed mutually adjacent and without gaps or overlaps to form a larger rectangle. One of the three rectangles has dimensions 70 by 110. Another of the rectangles has dimensions 40 by 80. What is the maximum perimeter of the third rectangle?

8. Priti is learning a new language called Tedio. During her one hour lesson, which started at midday, she looks at the clock and notices that the hour hand and the minute hand make exactly the same angle with the vertical, as shown in the diagram. How many whole seconds remain until the end of the lesson?
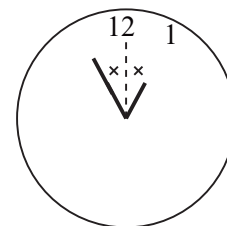
9. Robin shoots three arrows at a target. He earns points for each shot as shown in the figure. However, if any of his arrows miss the target or if any two of his arrows hit adjacent regions of the target, he scores a total of zero. How many different scores can he obtain?
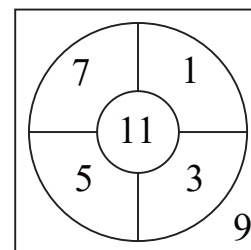
10. At each of the vertices of a cube sits a Bunchkin. Two Bunchkins are said to be adjacent if and only if they sit at either end of one of the cube's edges. Each Bunchkin is either a 'truther', who always tells the truth, or a 'liar', who always lies. All eight Bunchkins say 'I am adjacent to exactly two liars'. What is the maximum number of Bunchkins who are telling the truth?
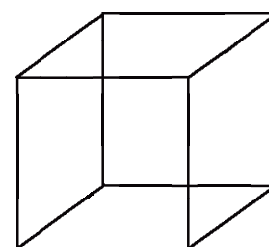
11. An infinite arithmetic progression of positive integers contains the terms 7, 11, 15, 71, 75 and 79. The first term in the progression is 7. Kim writes down all the possible values of the one-hundredth term in the progression. What is the sum of the numbers Kim writes down?

12. The pattern shown in the diagram is constructed using semicircles. Each semicircle has a diameter that lies on the horizontal axis shown and has one of the black dots at either end. The distance between each pair of adjacent black dots is 1 cm. The area, in cm$^2$, of the pattern that is shaded in grey is $\frac{1}{8}k\pi$. What is the value of $k$?
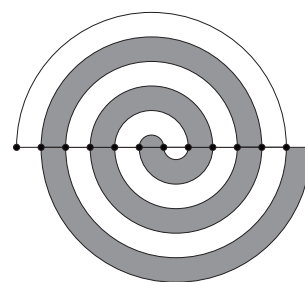
13. In the expression $\dfrac{k.a.n.g.a.r.o.o}{g.a.m.e}$, different letters stand for different non-zero digits but the same letter always stands for the same digit. What is the smallest possible integer value of the expression?

14. The set $S$ is given by $S = \{1, 2, 3, 4, 5, 6\}$. A non-empty subset $T$ of $S$ has the property that it contains no pair of integers that share a common factor other than 1. How many distinct possibilities are there for $T$?

15. Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct.
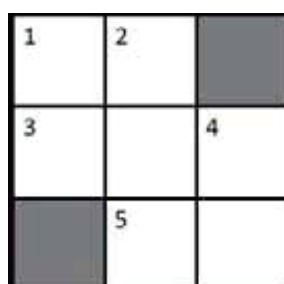


**ACROSS**
1. A square
3. The answer to this Kangaroo question
5. A square

**DOWN**
1. 4 down minus eleven
2. One less than a cube
4. The highest common factor of 1 down and 4 down is greater than one

16. The curve $x^2 + y^2 = 25$ is drawn. Points on the curve whose $x$-coordinate and $y$-coordinate are both integers are marked with crosses. All of those crosses are joined in turn to create a convex polygon $P$. What is the area of $P$?

17. Matthew writes a list of all three-digit squares backwards. For example, in his list Matthew writes the three-digit square '625' as '526'. Norma looks at Matthew's list and notices that some of the numbers are prime numbers. What is the mean of those prime numbers in Matthew's list?

18. The diagram shows a semicircle with diameter $PQ$ inscribed in a rhombus $ABCD$. The rhombus is tangent to the arc of the semicircle in two places. Points $P$ and $Q$ lie on sides $BC$ and $CD$ of the rhombus respectively. The line of symmetry of the semicircle is coincident with the diagonal $AC$ of the rhombus. It is given that $\angle CBA = 60°$. The semicircle has radius 10. The area of the rhombus can be written in the form $a\sqrt{b}$ where $a$ and $b$ are integers and $b$ is prime. What is the value of $ab + a + b$?
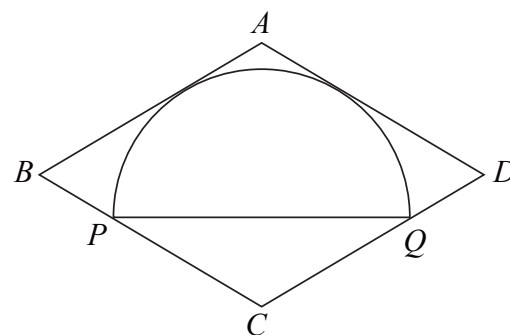


19. The sequence of functions $F_1(x)$, $F_2(x)$, ... satisfies the following conditions:

$$F_1(x) = x, \qquad F_{n+1}(x) = \frac{1}{1 - F_n(x)}.$$

The integer $C$ is a three-digit cube such that $F_C(C) = C$.
What is the largest possible value of $C$?

20. Let $a$, $b$ and $c$ be positive integers such that $a^2 = 2b^3 = 3c^5$. What is the minimum possible number of factors of $abc$ (including 1 and $abc$)?

1. Using this picture we can observe that
   1 + 3 + 5 + 7 = 4 × 4.
   What is the value of
   1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21?
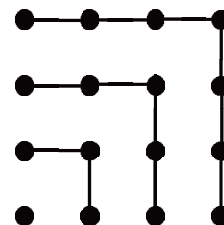
2. Both rows of the following grid have the same sum. What is the value of * ?

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 1050 |
|---|---|---|---|---|---|---|---|---|----|------|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | * |

3. Andrew has two containers for carrying water. The containers are cubes without tops and have base areas of 4 dm$^2$ and 36 dm$^2$ respectively. Andrew has to completely fill the larger cube with pond water, which must be carried from the pond using the smaller cube. What is the smallest number of visits Andrew has to make to the pond with the smaller cube?

4. How many four-digit numbers formed only of odd digits are divisible by five?

5. The notation $|x|$ is used to denote the absolute value of a number, regardless of sign. For example, $|7| = |-7| = 7$.
   The graphs $y = |2x| - 3$ and $y = |x|$ are drawn on the same set of axes. What is the area enclosed by them?

6.
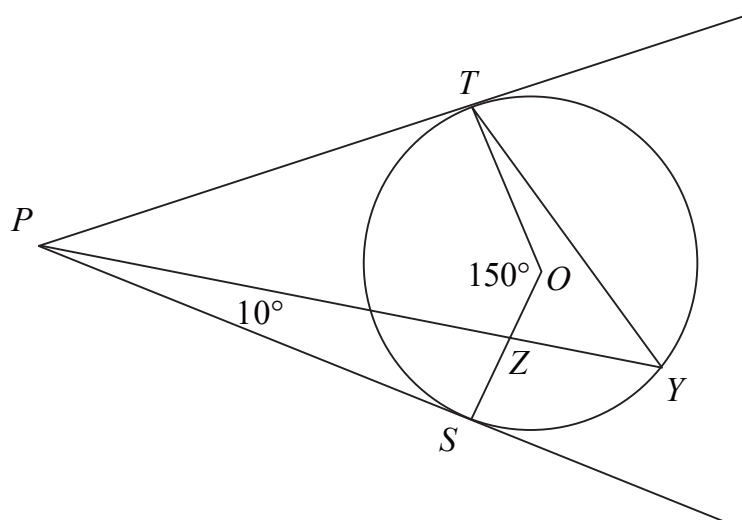
In the diagram, *PT* and *PS* are tangents to a circle with centre *O*. The point *Y* lies on the circumference of the circle; and the point *Z* is where the line *PY* meets the radius *OS*.

Also, ∠*SPZ* = 10° and ∠*TOS* = 150°.

How many degrees are there in the sum of ∠*PTY* and ∠*PYT*?

7. Bav is counting the edges on a particular prism. The prism has more than 310 edges, it has fewer than 320 edges and its number of edges is odd. How many edges does the prism have?

8. The real numbers $x$, $y$ and $z$ are a solution $(x, y, z)$ of the equation
   $(x^2 - 9)^2 + (y^2 - 4)^2 + (z^2 - 1)^2 = 0$. How many different possible values are there for
   $x + y + z$?

9. The diagram shows two concentric circles. Chord $AB$ of the larger circle is tangential to the smaller circle.

   The length of $AB$ is 32 cm and the area of the shaded region is $k\pi\,\text{cm}^2$.

   What is the value of $k$?
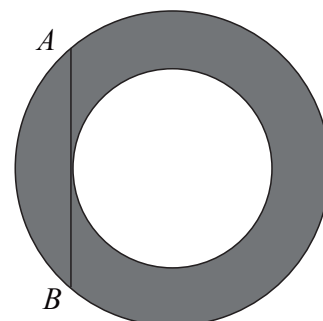
   

10. Consider the expression $1 * 2 * 3 * 4 * 5 * 6$.

    Each star in the expression is to be replaced with either '+' or '×'.

    $N$ is the largest possible value of the expression. What is the largest prime factor of $N$?

11. Stephanie enjoys swimming. She goes for a swim on a particular date if, and only if, the day, month (where January is replaced by '01' through to December by '12') and year are all of the same parity (that is they are all odd, or all are even). On how many days will she go for a swim in the two-year period between January 1st of one year and December 31st of the following year inclusive?

12. Delia is joining three vertices of a square to make four right-angled triangles.
    She can create four triangles doing this, as shown.

    

    How many right-angled triangles can Delia make by joining three vertices of a regular polygon with 18 sides?

13. This year, 2016, can be written as the sum of two positive integers $p$ and $q$ where $2p = 5q$ (as $2016 = 1440 + 576$). How many years between 2000 and 3000 inclusive have this property?

14. The lengths of the sides of a triangle are the integers $13, x, y$. It is given that $xy = 105$. What is the length of the perimeter of the triangle?

15. The large equilateral triangle shown consists of 36 smaller equilateral triangles. Each of the smaller equilateral triangles has area 10 cm$^2$.
The area of the shaded triangle is $K$ cm$^2$. Find $K$.
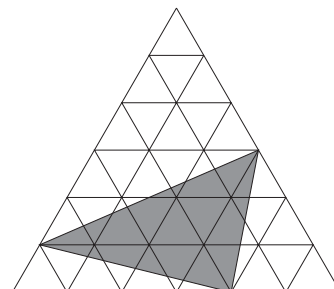
16. A function $f(x)$ has the property that, for all positive $x$, $3f(x) + 7f\left(\dfrac{2016}{x}\right) = 2x$.
What is the value of $f(8)$?

17. Students in a class take turns to practise their arithmetic skills. Initially a board contains the integers from 1 to 10 inclusive, each written ten times. On each turn a student first deletes two of the integers and then writes on the board the number that is one more than the sum of those two deleted integers. Turns are taken until there is only one number remaining on the board. Assuming no student makes a mistake, what is the remaining number?

18. The sum of the squares of four consecutive positive integers is equal to the sum of the squares of the next three consecutive integers. What is the square of the smallest of these integers?

19. Erin lists all three-digit primes that are 21 less than a square. What is the mean of the numbers in Erin's list?

20. A barcode of the type shown in the two examples is composed of alternate strips of black and white, where the leftmost and rightmost strips are always black. Each strip (of either colour) has a width of 1 or 2. The total width of the barcode is 12. The barcodes are always read from left to right. How many distinct barcodes are possible?

1.  In a pile of 200 coins, 2% are gold coins and the rest are silver. Simple Simon removes one silver coin every day until the pile contains 20% gold coins. How many silver coins does Simon remove?

2.  The value of the expression $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{5}}}$ is $\frac{a}{b}$, where $a$ and $b$ are integers whose only common factor is 1. What is the value of $a + b$?

3.  The diagram shows a solid with six triangular faces and five vertices. Andrew wants to write an integer at each of the vertices so that the sum of the numbers at the three vertices of each face is the same. He has already written the numbers 1 and 5 as shown.
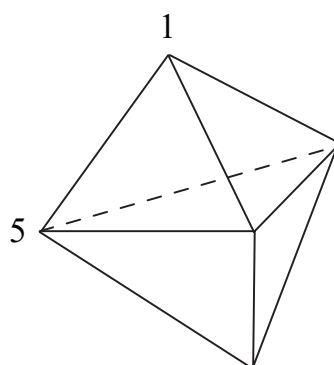
    

    What is the sum of the other three numbers he will write?

4.  A box contains two white socks, three blue socks and four grey socks. Rachel knows that three of the socks have holes in, but does not know what colour these socks are. She takes one sock at a time from the box without looking. How many socks must she take for her to be certain she has a pair of socks of the same colour without holes?

5.  The diagram shows two circles and a square with sides of length 10 cm. One vertex of the square is at the centre of the large circle and two sides of the square are tangents to both circles. The small circle touches the large circle. The radius of the small circle is $\left(a - b\sqrt{2}\right)$ cm.
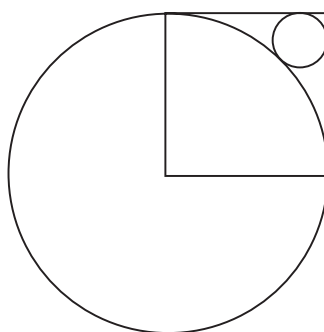
    

    What is the value of $a + b$?

6.  The median of a set of five positive integers is one more than the mode and one less than the mean. What is the largest possible value of the range of the five integers?

7.  The diagram shows a triangle *ABC* with area 12 cm². The sides of the triangle are extended to points *P*, *Q*, *R*, *S*, *T* and *U* as shown so that *PA* = *AB* = *BS*, *QA* = *AC* = *CT* and *RB* = *BC* = *CU*.
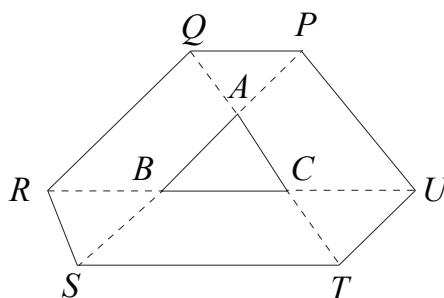


What is the area (in cm²) of hexagon *PQRSTU*?

8.  A mob of 2015 kangaroos contains only red and grey kangaroos. One grey kangaroo is taller than exactly one red kangaroo, one grey kangaroo is taller than exactly three red kangaroos, one grey kangaroo is taller than exactly five red kangaroos and so on with each successive grey kangaroo being taller than exactly two more red kangaroos than the previous grey kangaroo. The final grey kangaroo is taller than all the red kangaroos. How many grey kangaroos are in the mob?

9.  A large rectangle is divided into four identical smaller rectangles by slicing parallel to one of its sides. The perimeter of the large rectangle is 18 metres more than the perimeter of each of the smaller rectangles. The area of the large rectangle is 18 m² more than the area of each of the smaller rectangles. What is the perimeter in metres of the large rectangle?

10. Katherine and James are jogging in the same direction around a pond. They start at the same time and from the same place and each jogs at a constant speed. Katherine, the faster jogger, takes 3 minutes to complete one lap and first overtakes James 8 minutes after starting. How many seconds does it take James to complete one lap?

11. A ball is propelled from corner *A* of a *square* snooker table of side 2 metres. After bouncing off three cushions as shown, the ball goes into a pocket at *B*. The total distance travelled by the ball is $\sqrt{k}$ metres. What is the value of *k*?
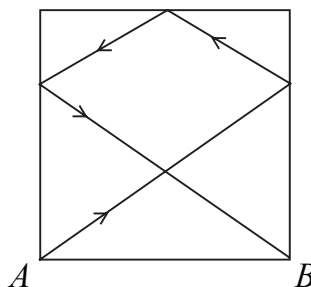


(Note that when the ball bounces off a cushion, the angle its path makes with the cushion as it approaches the point of impact is equal to the angle its path makes with the cushion as it moves away from the point of impact as shown in the diagram below.)

12. Chris planned a 210 km bike ride. However, he rode 5 km/h faster than he planned and finished his ride 1 hour earlier than he planned. His average speed for the ride was $x$ km/h. What is the value of $x$?

13. Twenty-five people who always tell the truth or always lie are standing in a queue. The man at the front of the queue says that everyone behind him always lies. Everyone else says that the person immediately in front of them always lies. How many people in the queue always lie?

14. Four problems were attempted by 100 contestants in a Mathematics competition. The first problem was solved by 90 contestants, the second by 85 contestants, the third by 80 contestants and the fourth by 75 contestants. What is the smallest possible number of contestants who solved all four problems?

15. The 5-digit number '$XX4XY$' is exactly divisible by 165. What is the value of $X + Y$?

16. How many 10-digit numbers are there whose digits are all 1, 2 or 3 and in which adjacent digits differ by 1?

17. In rectangle $JKLM$, the bisector of angle $KJM$ cuts the diagonal $KM$ at point $N$ as shown. The distances between $N$ and sides $LM$ and $KL$ are 8 cm and 1 cm respectively. The length of $KL$ is $(a + \sqrt{b})$ cm. What is the value of $a + b$?
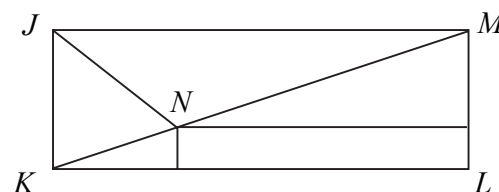
18. Numbers $a$, $b$ and $c$ are such that $\dfrac{a}{b + c} = \dfrac{b}{c + a} = \dfrac{c}{a + b} = k$.

    How many possible values of $k$ are there?

19. In quadrilateral $ABCD$, $\angle ABC = \angle ADC = 90°$, $AD = DC$ and $AB + BC = 20$ cm.
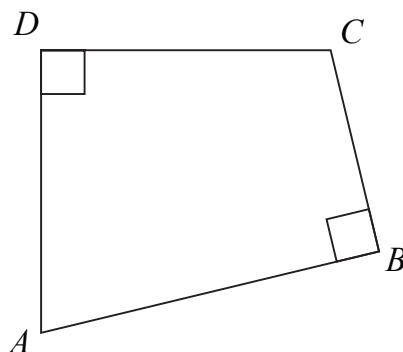
    What is the area in cm$^2$ of quadrilateral $ABCD$?

20. The number $N = 3^{16} - 1$ has a divisor of 193. It also has some divisors between 75 and 85 inclusive. What is the sum of these divisors?

1. Three standard dice are stacked in a tower so that the numbers on each pair of touching faces add to 5. The number on the top of the tower is even. What is the number on the base of the tower?

2. How many prime numbers $p$ have the property that $p^4 + 1$ is also prime?

3. Neil has a combination lock. He knows that the combination is a four-digit number with first digit 2 and fourth digit 8 and that the number is divisible by 9. How many different numbers with that property are there?

4. In the diagram, triangle $ABC$ is isosceles with $CA = CB$ and point $D$ lies on $AB$ with $AD = AC$ and $DB = DC$. What is the size in degrees of angle $BCA$?
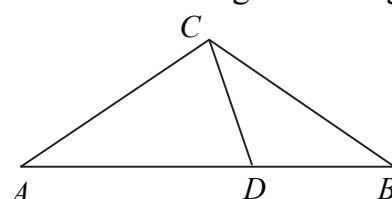


5. Six of the seven numbers 11, 20, 15, 25, 16, 19 and 17 are divided into three groups of two numbers so that the sum of the two numbers in each group is the same. Which number is not used?

6. The numbers $x$, $y$ and $z$ satisfy the equations $x^2yz^3 = 7^3$ and $xy^2 = 7^9$. What is the value of $\dfrac{xyz}{7}$?

7. A table of numbers has 21 columns labelled 1, 2, 3, ..., 21 and 33 rows labelled 1, 2, 3, ..., 33. Every element of the table is equal to 2. All the rows whose label is not a multiple of 3 are erased. All the columns whose label is not an even number are erased. What is the sum of the numbers that remain in the table?

8. Andrew wishes to place a number in each circle in the diagram. The sum of the numbers in the circles of any closed loop of length three must be 30. The sum of the numbers in the circles of any closed loop of length four must be 40. He places the number 9 in the circle marked $X$. What number should he put in the circle marked $Y$?
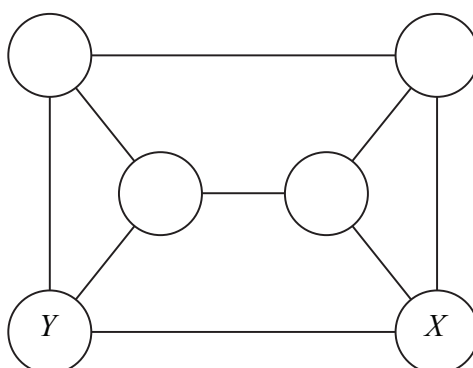
9. Each of the cubes in the diagram has side length 3 cm. The length of $AB$ is $\sqrt{k}$ cm. What is the value of $k$?
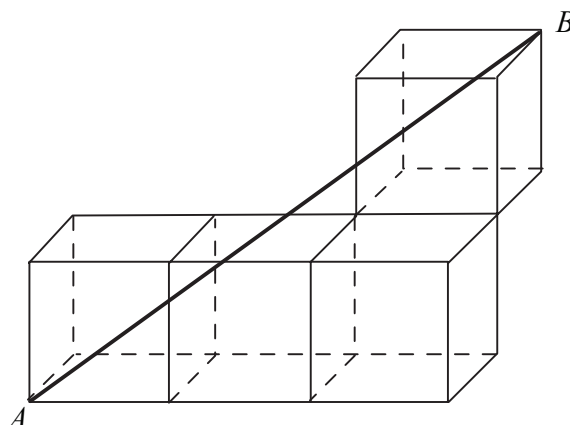


10. A Mathematical Challenge consists of five problems, each of which is worth a different whole number of marks. Carl solved all five problems correctly. He scored 10 marks for the two problems with the lowest numbers of marks and 18 marks for the two problems with the highest numbers of marks. How many marks did he score for all five problems?

11. The mean weight of five children is 45 kg. The mean weight of the lightest three children is 42 kg and the mean weight of the heaviest three children is 49 kg. What is the median weight of the children in kg?

12. On Old MacDonald's farm, the numbers of horses and cows are in the ratio 6:5, the numbers of pigs and sheep are in the ratio 4:3 and the numbers of cows and pigs are in the ratio 2:1. What is the smallest number of animals that can be on the farm?

13. The diagram shows a circle with diameter $AB$. The coordinates of $A$ are $(-2, 0)$ and the coordinates of $B$ are $(8, 0)$. The circle cuts the $y$-axis at points $D$ and $E$. What is the length of $DE$?
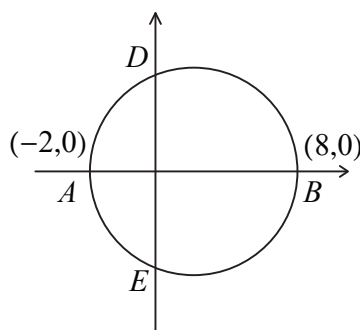


14. Rachel draws 36 kangaroos using three different colours. 25 of the kangaroos are drawn using some grey, 28 are drawn using some pink and 20 are drawn using some brown. Five of the kangaroos are drawn using all three colours. How many kangaroos did she draw that use only one colour?

15. A box contains seven cards numbered from 301 to 307. Graham picks three cards from the box and then Zoe picks two cards from the remainder. Graham looks at his cards and then says "I know that the sum of the numbers on your cards is even". What is the sum of the numbers on Graham's cards?

16. The numbers $x$, $y$ and $z$ satisfy the equations $x + y + z = 15$ and $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 0$. What is the value of $x^2 + y^2 + z^2$?

17. In the diagram, $PQRS$ is a square. $M$ is the midpoint of $PQ$. The area of the square is $k$ times the area of the shaded region. What is the value of $k$?
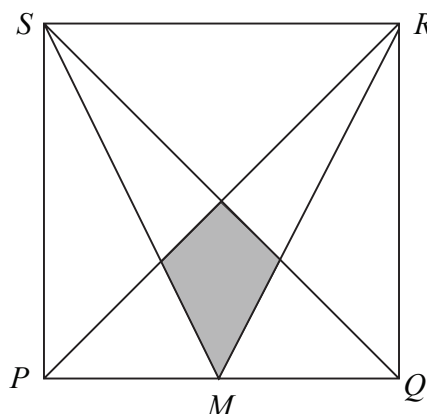


18. Twenty-five workmen have completed a fifth of a project in eight days. Their foreman then decides that the project must be completed in the next 20 days. What is the smallest number of additional workmen required to complete the project on time?

19. In the long multiplication sum shown, each asterisk stands for one digit.

What is the sum of the digits of the answer?

$$
\begin{array}{r}
*** \\
\times \ *** \\
\hline
22** \\
90*0 \\
**2** \\
\hline
56*** \\
\end{array}
$$

20. In the quadrilateral $PQRS$ with $PQ = PS = 25$ cm and $QR = RS = 15$ cm, point $T$ lies on $PQ$ so that $PT = 15$ cm and so that $TS$ is parallel to $QR$. What is the length in centimetres of $TS$?
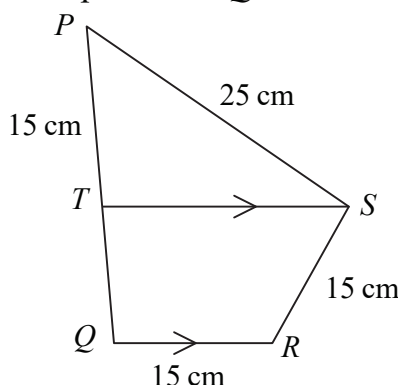
1. Adam, Bill and Carl have 30 sweets between them. Bill gives 5 sweets to Carl, Carl gives 4 sweets to Adam and Adam gives 2 sweets to Bill. Now each of them has the same number of sweets. How many sweets did Carl have initially?

2. An $i$-rectangle is defined to be a rectangle all of whose sides have integer length. Two $i$-rectangles are considered to be the same if they have the same side-lengths. The sum of the areas of all the different $i$-rectangles with perimeter 22 cm is $A$ cm$^2$. What is the value of $A$?

3. Some historians claim that the ancient Egyptians used a rope with two knots tied in it to construct a right-angled triangle by joining the two ends of the rope and taking the vertices of the triangle to be at the two knots and at the join. The length of the rope shown is 60 m and one of the knots is at $X$, which is 15 m from one end of the rope. How many metres from the other end of the rope should the second knot be placed to be able to create a right-angled triangle with the right angle at $X$?
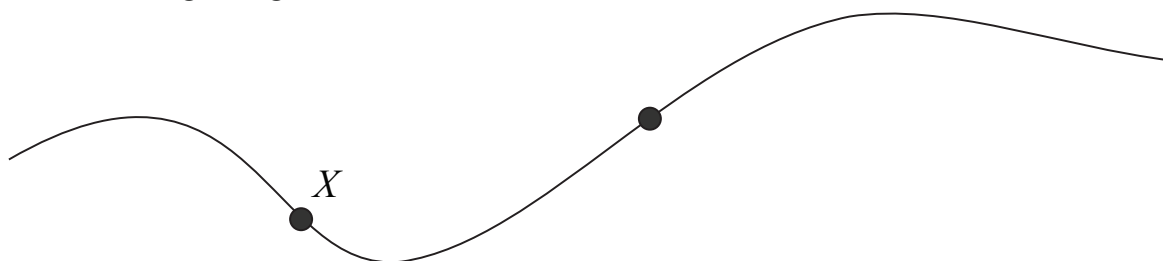


4. The height, width and length of a cube are multiplied by 2, 3 and 6 respectively to create a cuboid. The surface area of the cuboid is $N$ times the surface area of the original cube. What is the value of $N$?

5. In a university admissions test, Dean gets exactly 10 of the first 15 questions correct. He then answers all the remaining questions correctly. Dean finds out he has answered 80% of all the questions correctly. How many questions are there on the test?

6. In the diagram, $AE$ is divided into four equal parts and semicircles have been drawn with $AE$, $AD$ and $DE$ as diameters. This has created two new paths, an upper path and a lower path, from $A$ to $E$. The ratio of the length of the upper path to the length of the lower path can be written as $a : b$ in its lowest terms. What is the value of $a + b$?
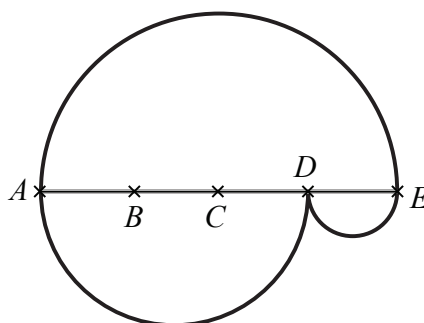
7.  A mathematically skilful spider has spun a web and the lengths of some of the strands (which are all straight lines) are as shown in the diagram. It is known that $x$ is an integer. What is the value of $x$?
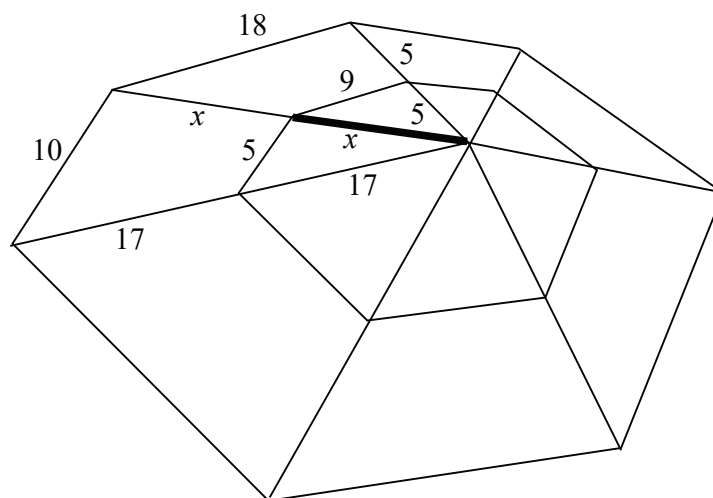


8.  The square $ABCD$ has sides of length 1. All possible squares that share two vertices with $ABCD$ are drawn. The boundary of the region formed by the union of these squares is an irregular polygon. What is the area of this polygon?
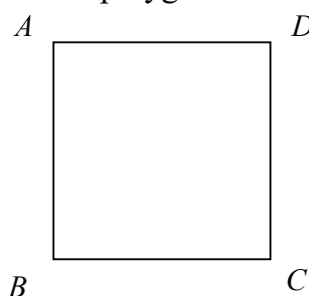


9.  In triangle $ABC$, angle $B$ is 25% smaller than angle $C$ and 50% larger than angle $A$. What is the size in degrees of angle $B$?

10. In the equation $2^{m+1} + 2^m = 3^{n+2} - 3^n$, $m$ and $n$ are integers. What is the value of $m$?

11. The diagram shows two semicircles. The chord $CD$ of the larger semicircle is parallel to $AB$, and touches the smaller semicircle. The length of $CD$ is 32 m. The area of the shaded region is $k\pi$ m$^2$. What is the value of $k$?
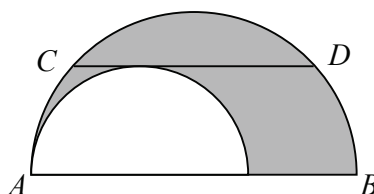


12. The sum of five consecutive integers is equal to the sum of the next three consecutive integers. What is the largest of these eight integers?

13. Zoe was born on her mother's 24th birthday so they share birthdays. Assuming they both live long lives, on how many birthdays will Zoe's age be a factor of her mother's age?

14. What is the largest three-digit integer that can be written in the form $n + \sqrt{n}$ where $n$ is an integer?

15. How many integers $a$ are there for which the roots of the quadratic equation $x^2 + ax + 2013 = 0$ are integers?

16. A sphere of radius 3 has its centre at the origin. How many points on the surface of the sphere have coordinates that are all integers?

17. The length of each side of the rhombus $PQRS$ is equal to the geometric mean of the lengths of its diagonals. What is the size in degrees of the obtuse angle $PQR$?
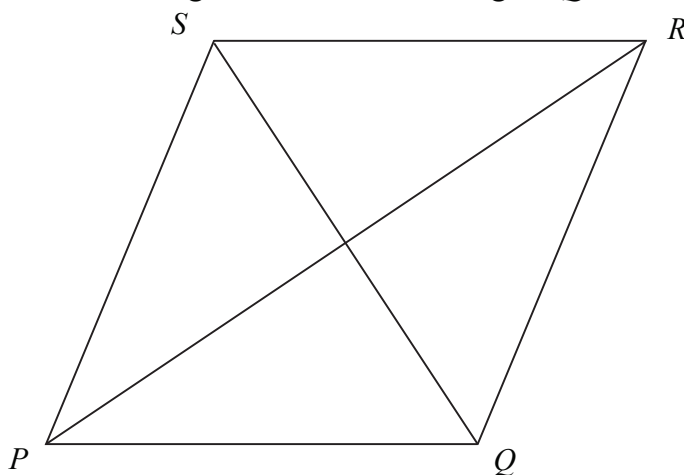


[ The geometric mean of 2 values $x_1$ and $x_2$ is given by $\sqrt{x_1 x_2}$. ]

18. How many of the first 2013 triangular numbers are multiples of 5?

19. The increasing sequence 1, 3, 4, 9, 10, 12, 13, … contains all the powers of 3 and all the numbers that can be written as the sum of two or more distinct powers of 3. What is the 70th number in the sequence?

20. Rachel and Nicky stand at either end of a straight track. They then run at constant (but different) speeds to the other end of the track, turn and run back to their original end at the same speed they ran before. On their first leg, they pass each other 20 m from one end of the track. When they are both on their return leg, they pass each other for a second time 10 m from the other end of the track. How many metres long is the track?

1. How many zeroes are there at the end of the number which is the product of the first 2012 prime numbers?

2. The size of the increase from each term to the next in the list $a$, $225\frac{1}{2}$, $c$, $d$, $284$ is always the same. What is the value of $a$?

3. On the grid shown in the diagram, the shaded squares form a region, $A$.

   What is the maximum number of additional grid squares which can be shaded to form a region $B$ such that $B$ contains $A$ and that the lengths of the perimeters of $A$ and $B$ are the same?
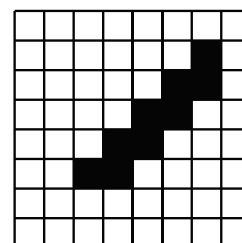
   

4. Five cards are laid on a table, as shown. Every card has a letter on one side and a number on the other side.

   Peter says: "For every card on the table, if there is a vowel on one side of the card, then there is an even number on the other side."

   What is the smallest number of cards Sylvia must turn over in order to be certain that Peter is telling the truth?
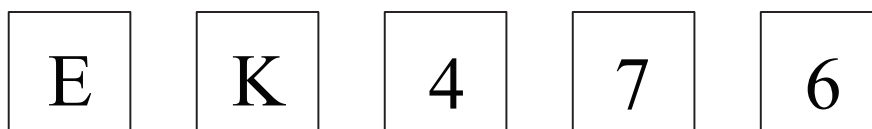
   

5. Susan has two pendants made of the same material. They are equally thick and weigh the same. The first pendant is in the shape of an annulus created from two concentric circles, with diameters 8 cm and 12 cm, as shown. The shape of the second pendant is a disc. The diameter of the second pendant is written in the form $a\sqrt{b}$, where $a$ is an integer and $b$ is a prime integer.

   What is the value of $a + b$?
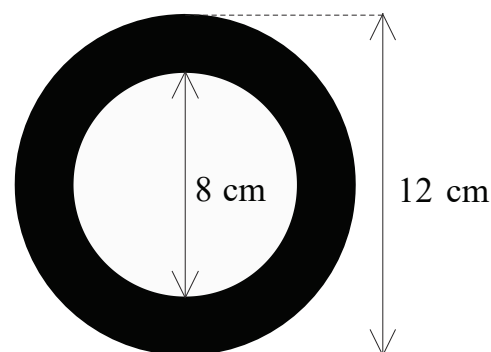
   

6. Given that $4^x = 9$ and $9^y = 256$, what is the value of $xy$?

7. When 1001 is divided by a single-digit number, the remainder is 5.
   What is the remainder when 2012 is divided by the same single-digit number?

8. The three prime numbers $a$, $b$ and $c$ are such that $a > b > c$, $a + b + c = 52$ and $a - b - c = 22$.
   What is the value of $abc$?

9.  The diagram shows a circle touching a sector of another circle in three places. The ratio of the radius of the sector to the radius of the small circle is 3:1. The ratio of the area of the sector to the area of the small circle, written in its simplest form, is $p : q$.

    What is the value of $p + q$?
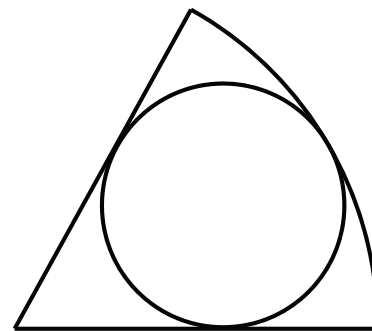
10. Sixteen teams play in a volleyball league. Each team plays one game against every other team. For each game, the winning team is awarded 1 point, and the losing team 0 points. There are no draws. After all the games have been played and the teams have been ranked according to their total scores, the total scores form a sequence where the difference between consecutive terms is constant.

    How many points did the team in first place receive?

11. Last year there were 30 more boys than girls in the school choir. This year the number of choir members has increased by 10%, the number of girls has increased by 20% and the number of boys by 5%.

    How many members does the choir have this year?

12. The cells of a 4 × 4 grid are coloured black and white as shown in Figure 1. One move allows us to exchange the colourings of any two cells positioned in the same row or in the same column.

    What is the minimum number of moves needed to obtain Figure 2?
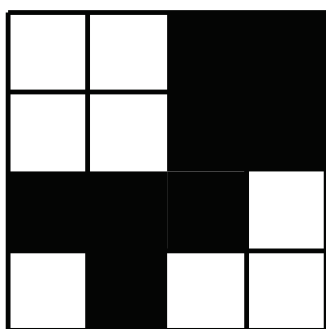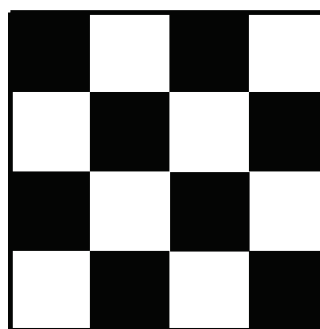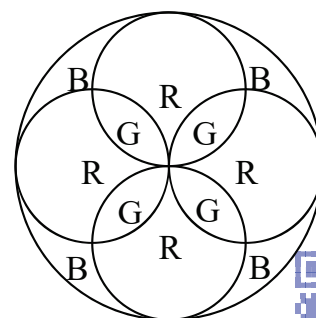
            Figure 1                                    Figure 2

13. A circular stained-glass window is shown in the diagram. The four smaller circles are the same size and are positioned at equal intervals around the centre of the large circle. The letters R, G and B have been placed in regions of red, green and blue glass respectively. The total area of the green glass is 400.

    What is the area of the blue glass?

14. The diagram shows a cuboid. In triangle $XYZ$, the lengths of $XY$, $XZ$ and $YZ$ are 9, 8 and $\sqrt{55}$ respectively.

    What is the length of the diagonal $XA$ shown?
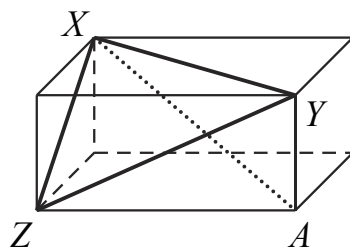


15. The equation $x^2 - bx + 80 = 0$, where $b > 0$, has two integer-valued solutions. What is the sum of the possible values of $b$?

16. Given that $a + b = 5$ and $ab = 3$, what is the value of $a^4 + b^4$?

17. David removed one number from ten consecutive natural numbers. The sum of the remaining numbers was 2012.

    Which number did he remove?

18. The diagram shows a square divided into six smaller squares labelled $A$, $B$, $C$, $D$, $E$ and $F$. Two squares are considered to be adjacent if they have more than one point in common. The numbers 1, 2, 3, 4, 5 and 6 are to be placed in the smaller squares, one in each, so that no two adjacent squares contain numbers differing by 3.
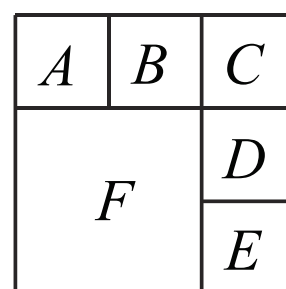
    How many different arrangements are possible?



19. A rectangle which has integer-length sides and area 36 is cut from a square with sides of length 20 so that one of the sides of the rectangle forms part of one of the sides of the square.

    What is the largest possible perimeter of the remaining shape?

20. How many subsets of the set $\{1, \ 2, \ 3, \ 4, \ 5, \ 6, \ 7, \ 8, \ 9, \ 10\}$ exist in which the sum of the largest element and the smallest element is 11?
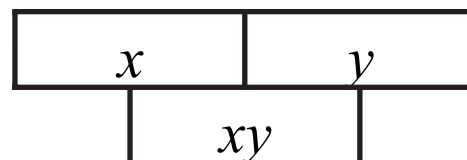
1. The diagram below is to be completed so that:
   a) each cell contains a positive integer;
   b) apart from the top row, the number in each cell is the product of the numbers in the two cells immediately above;
   c) the six numbers are all different.

   What is the smallest possible total of the six numbers?
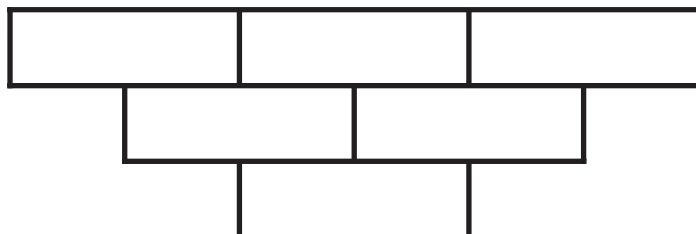
   RULE:

2. The mean number of students accepted by a school in the four years 2007 to 2010 was 325. The mean number of students accepted by the school in the five years 2007 to 2011 was 4% higher.

   How many students did this school accept in 2011?

3. 200 people stand in a line. The prize-giver walks along the line 200 times, always starting at the same end. On the first pass, the prize-giver gives each person a pound coin. On the second pass along the line, the prize-giver gives every second person another pound. On the third pass, every third person is given another pound, and so on.

   After 200 passes, how many pounds has the 120th person been given?

4. The diagram below includes two squares: one has sides of length 20 and the other has sides of length 10.

   What is the area of the shaded region?
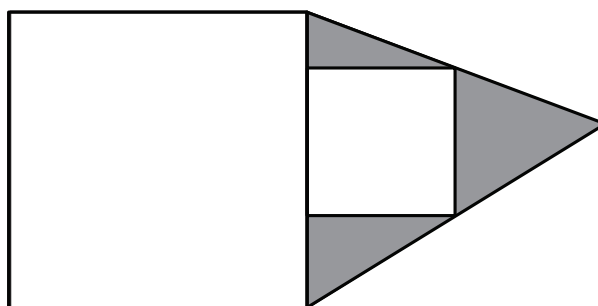
5. How many positive two-digit numbers are there whose square and cube both end in the same digit?

6. The lengths of two sides of an acute-angled triangle and the perpendicular height from the third side of the triangle are 12, 13 and 15 (possibly not in that order).

   What is the area of the triangle?

7. In the diagram, the radius of the circle is equal to the length *AB*.
   What is the size of angle *ACB*, in degrees?
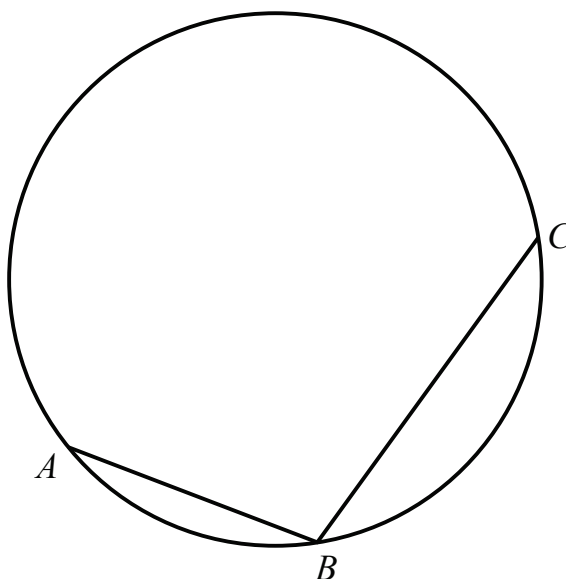


8. The price of an item in pounds and pence is increased by 4%. The new price is exactly *n* pounds where *n* is a whole number.
   What is the smallest possible value of *n*?

9. How many squares have $(-1, -1)$ as a vertex and at least one of the coordinate axes as an axis of symmetry?

10. What is the value of $\left(\sqrt{8 + 2\sqrt{7}} - \sqrt{8 - 2\sqrt{7}}\right)^2$?

11. In the diagram, *ABC* is a triangle with sides $AB = 13$, $BC = 12$ and $AC = 5$. The point *D* is any point inside the triangle with $CD = 4$ and the perpendicular distances from *D* to the sides of the triangle are $e, f$ and $g$, as shown.
    What is the value of $5e + 12f + 13g$?
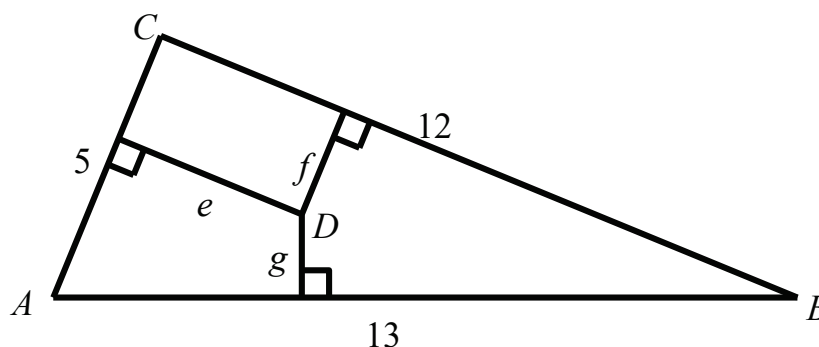


12. Elections in Herbville were held recently. Everyone who voted for the Broccoli Party had already eaten broccoli. Of those who voted for other parties, 90% had never eaten broccoli. Of those who voted, 46% had eaten broccoli.
    What percentage voted for the Broccoli Party?

13. A manager in a store has to determine the price of a sweater. Market research gives him the following data: If the price is €75, then 100 teenagers will buy the sweater. Each time the price is increased by €5, 20 fewer teenagers will buy the sweater. However, each time the price is decreased by €5, 20 sweaters more will be sold. The sweaters cost the company €30 apiece.
What is the sale price that maximizes profits?

14. The diagram shows a rectangle $ABCD$ with $AB = 16$ and $BC = 12$. Angle $ACE$ is a right angle and $CE = 15$. The line segments $AE$ and $CD$ meet at $F$.
What is the area of triangle ACF?
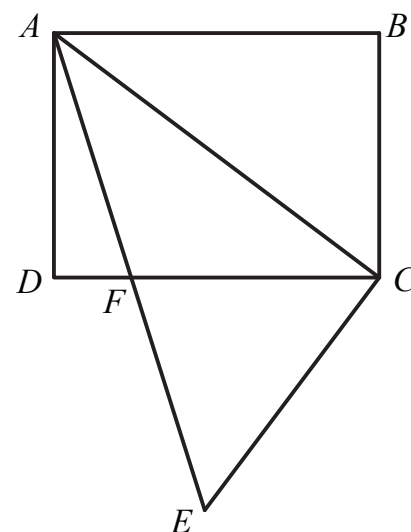


15. For each real number $x$, let $f(x)$ be the minimum of the numbers $3x + 1$, $2x + 3$ and $-4x + 24$.
What is the maximum value of $f(x)$ ?

16. The integer $m$ has ninety-nine digits, all of them nines.
What is the sum of the digits of $m^2$ ?

17. In rectangle $ABCD$, the midpoints of sides $BC$, $CD$ and $DA$ are $P$, $Q$ and $R$ respectively. The point $M$ is the midpoint of $QR$. The area of triangle $APM$ is a fraction $m/n$ of the area of rectangle $ABCD$, where $m$ and $n$ are integers and $m/n$ is in its simplest form.
What is the value of $m + n$?

18. The integers $a$, $b$ and $c$ are such that $0 < a < b < c < 10$. The sum of all three-digit numbers that can be formed by a permutation of these three integers is 1554.
What is the value of $c$?

19. Given that $\left(a + \dfrac{1}{a}\right)^2 = 6$ and $a^3 + \dfrac{1}{a^3} = N\sqrt{6}$ and $a > 0$, what is the value of $N$?

20. The polynomial $f(x)$ is such that $f(x^2 + 1) \equiv x^4 + 4x^2$ and $f(x^2 - 1) \equiv ax^4 + 4bx^2 + c$.
What is the value of $a^2 + b^2 + c^2$?

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