1 Find the binomial expansion of each of the following in ascending powers of x up to and including the term in x^3, for $|x| < 1$.

a $(1+x)^{-1}$ **b** $(1+x)^{\frac{1}{2}}$ **c** $2(1+x)^{-3}$ **d** $(1+x)^{\frac{2}{3}}$

e $\sqrt[3]{1-x}$ **f** $\dfrac{1}{(1+x)^2}$ **g** $\dfrac{1}{4(1-x)^4}$ **h** $\dfrac{3}{\sqrt{1-x}}$

2 Expand each of the following in ascending powers of x up to and including the term in x^3 and state the set of values of x for which each expansion is valid.

a $(1+2x)^{\frac{1}{2}}$ **b** $(1-3x)^{-1}$ **c** $(1-4x)^{-\frac{1}{2}}$ **d** $(1+\frac{1}{2}x)^{-3}$

e $(1-6x)^{\frac{1}{3}}$ **f** $(1+\frac{1}{4}x)^{-4}$ **g** $(1+2x)^{\frac{3}{2}}$ **h** $(1-3x)^{-\frac{4}{3}}$

3 **a** Expand $(1-2x)^{\frac{1}{2}}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3.

b By substituting a suitable value of x in your expansion, find an estimate for $\sqrt{0.98}$

c Show that $\sqrt{0.98} = \frac{7}{10}\sqrt{2}$ and hence find the value of $\sqrt{2}$ correct to 8 significant figures.

4 Expand each of the following in ascending powers of x up to and including the term in x^3 and state the set of values of x for which each expansion is valid.

a $(2+x)^{-1}$ **b** $(4+x)^{\frac{1}{2}}$ **c** $(3-x)^{-3}$ **d** $(9+3x)^{\frac{1}{2}}$

e $(8-24x)^{\frac{1}{3}}$ **f** $(4-3x)^{-1}$ **g** $(4+6x)^{-\frac{1}{2}}$ **h** $(3+2x)^{-2}$

5 **a** Expand $(1+2x)^{-1}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3.

b Hence find the series expansion of $\dfrac{1-x}{1+2x}$, $|x| < \frac{1}{2}$, in ascending powers of x up to and including the term in x^3.

6 Find the first four terms in the series expansion in ascending powers of x of each of the following and state the set of values of x for which each expansion is valid.

a $\dfrac{1+3x}{1-x}$ **b** $\dfrac{2x-1}{(1+4x)^2}$ **c** $\dfrac{3+x}{2-x}$ **d** $\dfrac{1-x}{\sqrt{1+2x}}$

7 **a** Express $\dfrac{x-2}{(1-x)(1-2x)}$ in partial fractions.

b Hence find the series expansion of $\dfrac{x-2}{(1-x)(1-2x)}$ in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid.

8 By first expressing f(x) in partial fractions, find the series expansion of f(x) in ascending powers of x up to and including the term in x^3 and state the set of values of x for which it is valid.

a $f(x) \equiv \dfrac{4}{(1+x)(1-3x)}$ **b** $f(x) \equiv \dfrac{1-6x}{1+3x-4x^2}$ **c** $f(x) \equiv \dfrac{5}{2-3x-2x^2}$

d $f(x) \equiv \dfrac{7x-3}{x^2-4x+3}$ **e** $f(x) \equiv \dfrac{3+5x}{(1+3x)(1+x)^2}$ **f** $f(x) \equiv \dfrac{2x^2+4}{2x^2+x-1}$

1 **a** Expand $(1 - x)^{\frac{1}{2}}$, $|x| < 1$, in ascending powers of x up to and including the term in x^3.

b By substituting $x = 0.01$ in your expansion, find the value of $\sqrt{11}$ correct to 9 significant figures.

2 The series expansion of $(1 + 8x)^{\frac{1}{2}}$, in ascending powers of x up to and including the term in x^3, is

$$1 + 4x + ax^2 + bx^3, \ |x| < \tfrac{1}{8}.$$

a Find the values of the constants a and b.

b Use the expansion, with $x = 0.01$, to find the value of $\sqrt{3}$ to 5 decimal places.

3 **a** Expand $(9 - 6x)^{\frac{1}{2}}$, $|x| < \frac{3}{2}$, in ascending powers of x up to and including the term in x^3, simplifying the coefficient in each term.

b Use your expansion with a suitable value of x to find the value of $\sqrt{8.7}$ correct to 7 significant figures.

4 **a** Expand $(1 + 6x)^{\frac{1}{3}}$, $|x| < \frac{1}{6}$, in ascending powers of x up to and including the term in x^3.

b Use your expansion, with $x = 0.004$, to find the cube root of 2 correct to 7 significant figures.

5 **a** Expand $(1 + 2x)^{-3}$ in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid.

b Hence, or otherwise, find the series expansion in ascending powers of x up to and including the term in x^3 of $\dfrac{1+3x}{(1+2x)^3}$.

6 Find the coefficient of x^2 in the series expansion of $\dfrac{2+x}{\sqrt{4-2x}}$, $|x| < 2$.

7 **a** Find the values of A and B such that

$$\frac{2-11x}{1-5x+4x^2} \equiv \frac{A}{1-x} + \frac{B}{1-4x}.$$

b Hence, find the series expansion of $\dfrac{2-11x}{1-5x+4x^2}$ in ascending powers of x up to and including the term in x^3 and state the set of values of x for which the expansion is valid.

8 $$f(x) \equiv \frac{4-17x}{(1+2x)(1-3x)^2}, \ |x| < \tfrac{1}{3}.$$

a Express $f(x)$ in partial fractions.

b Hence, or otherwise, find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3.

9 The first three terms in the expansion of $(1 + ax)^b$, in ascending powers of x, for $|ax| < 1$, are

$$1 - 6x + 24x^2.$$

a Find the values of the constants a and b.

b Find the coefficient of x^3 in the expansion.