1 Give a counter-example to prove that each of the following statements is false.

a If $a^2 - b^2 > 0$, where a and b are real, then $a - b > 0$.

b There are no prime numbers divisible by 7.

c If x and y are irrational and $x \neq y$, then xy is irrational.

d For all real values of x, $\cos(90 - |x|)^\circ = \sin x^\circ$.

2 For each statement, either prove that it is true or find a counter-example to prove that it is false.

a There are no prime numbers divisible by 6.

b $(3^n + 2)$ is prime for all positive integer values of n.

c $\sqrt{n}$ is irrational for all positive integers n.

d If a, b and c are integers such that a is divisible by b and b is divisible by c, then a is divisible by c.

3 Use proof by contradiction to prove each of the following statements.

a If n^3 is odd, where n is a positive integer, then n is odd.

b If x is irrational, then $\sqrt{x}$ is irrational.

c If a, b and c are integers and bc is not divisible by a, then b is not divisible by a.

d If $(n^2 - 4n)$ is odd, where n is a positive integer, then n is odd.

e There are no positive integers, m and n, such that $m^2 - n^2 = 6$.

4 Given that x and y are integers and that $(x^2 + y^2)$ is divisible by 4, use proof by contradiction to prove that

a x and y are not both odd,

b x and y are both even.

5 For each statement, either prove that it is true or find a counter-example to prove that it is false.

a If a and b are positive integers and $a \neq b$, then $\log_a b$ is irrational.

b The difference between the squares of any two consecutive odd integers is divisible by 8.

c $(n^2 + 3n + 13)$ is prime for all positive integer values of n.

d For all real values of x and y, $x^2 - 2y(x - y) \geq 0$.

6 **a** Prove that if

$$\sqrt{2} = \frac{p}{q},$$

where p and q are integers, then p must be even.

b Use proof by contradiction to prove that $\sqrt{2}$ is irrational.