1 For each of the following geometric series, write down the common ratio and find the value of the eighth term.

a $3 + 9 + 27 + 81 + \ldots$ **b** $1024 + 256 + 64 + 16 + \ldots$ **c** $1 - 2 + 4 - 8 + \ldots$

2 For each of the following geometric series, find an expression for the nth term.

a $1 + 5 + 25 + 125 + \ldots$ **b** $3 - 12 + 48 - 192 + \ldots$ **c** $81 + 54 + 36 + 24 + \ldots$

3 Find the sum of the first 12 terms of each of the following geometric series.

a $2 + 4 + 8 + 16 + \ldots$ **b** $640 + 320 + 160 + 80 + \ldots$ **c** $\frac{1}{6} - \frac{1}{2} + 1\frac{1}{2} - 4\frac{1}{2} + \ldots$

4 Given the first term, a, the common ratio, r, and the number of terms, n, find the sum of each of the following geometric series. Give your answers to 3 decimal places where appropriate.

a $a = 4$, $r = 3$, $n = 8$ **b** $a = 48$, $r = \frac{1}{2}$, $n = 14$ **c** $a = -1$, $r = -4$, $n = 12$

d $a = 200$, $r = 0.7$, $n = 20$ **e** $a = 120$, $r = -\frac{3}{4}$, $n = 15$ **f** $a = -25$, $r = 1.2$, $n = 30$

5 Evaluate to an appropriate degree of accuracy

a $\sum_{r=1}^{9} 3^r$ **b** $\sum_{r=1}^{6} 8^{r+1}$ **c** $\sum_{r=1}^{10} (10\times 2^r)$ **d** $\sum_{r=1}^{8} (0.8)^r$

e $\sum_{r=1}^{10} \left[12\times(\frac{1}{6})^r\right]$ **f** $\sum_{r=1}^{9} (-4)^r$ **g** $\sum_{r=4}^{20} (\frac{1}{2})^r$ **h** $\sum_{r=3}^{9} \left[2\times(-3)^r\right]$

6 The second and third terms of a geometric series are 2 and 10 respectively.

a Find the common ratio of the series.

b Find the first term of the series.

c Find the sum of the first eight terms of the series.

7 The first and fourth terms of a geometric series are 2 and 54 respectively.

a Find the common ratio of the series.

b Find the ninth term of the series.

8 The third and fourth terms of a geometric series are 24 and 8 respectively.

a Find the common ratio of the series.

b Find the first term of the series.

c Find, to 3 decimal places, the sum of the first 11 terms of the series.

9 The first and third terms of a geometric series are 6 and 24 respectively.

a Find the two possible values for the common ratio of the series.

Given also that the common ratio of the series is positive,

b find the sum of the first 15 terms of the series.

10 The first and fourth terms of a geometric series are 768 and −96 respectively.

a Find the common ratio of the series.

b Find the tenth term of the series.

11 The second and fifth terms of a geometric series are 0.5 and 32 respectively.

a Find the first term and common ratio of the series.

b Find the number of terms of the series that are smaller than 10 000.

12 The sum of the first four terms of a geometric series is 130 and its common ratio is $1\frac{1}{2}$.

a Find the first term of the series.

b Find the eighth term of the series.

c Find the least value of n for which the sum of the first n terms of the series is greater than 30 000.

13 All the terms of a geometric series are positive. The sum of the first and second terms of the series is 10.8 and the sum of the third and fourth terms of the series is 43.2

a Find the first term and common ratio of the series.

b Find the sum of the first 16 terms of the series.

14 For each of the following geometric series, either find its sum to infinity or explain why this cannot be found.

a $12 + 6 + 3 + 1.5 + \ldots$ **b** $270 + 90 + 30 + 10 + \ldots$ **c** $25 - 30 + 36 - 43.2 + \ldots$

d $216 + 144 + 96 + 64 + \ldots$ **e** $\frac{8}{25} + \frac{2}{5} + \frac{1}{2} + \frac{5}{8} + \ldots$ **f** $500 - 300 + 180 - 108 + \ldots$

15 Find the sum to infinity of the geometric series with nth term

a $(0.9)^n$ **b** $6 \times (\frac{1}{2})^n$ **c** $(-\frac{3}{4})^{n-1}$ **d** $40 \times (0.8)^n$

16 A geometric series has first term 80 and common ratio 0.2

a Find the sum to infinity of the series.

b Find the difference between the sum to infinity of the series and the sum of the first six terms of the series.

17 A sequence is defined by the recurrence relation

$$u_{n+1} = \tfrac{1}{3}u_n, \quad n > 0, \quad u_1 = 1.$$

a Write down the first four terms of the sequence.

b Evaluate $\sum_{r=1}^{\infty} u_r$.

18 The common ratio of a geometric series is 0.55 and the sum to infinity of the series is 40.

a Find the first term of the series.

b Find the smallest value of n for which the nth term of the series is less than 0.001

19 The sum, S_n, of the first n terms of a geometric series is given by $S_n = 2^n - 1$.

a Find the first term and the fifth term of the series.

b Find an expression for the nth term of the series.

20 The first three terms of a geometric series are $(k + 10)$, k and $(k - 6)$ respectively.

a Find the value of the constant k.

b Find the sum to infinity of the series.

1 The third and fourth terms of a geometric series are 27 and $20\frac{1}{4}$ respectively.

a Find the first term of the series.

b Find the sum to infinity of the series.

2 The first three terms of a geometric series are $(k - 8)$, $(k + 4)$ and $(3k + 2)$ respectively, where k is a positive constant.

a Find the value of k.

b Find the sixth term of the series.

c Show that the sum of the first ten terms of the series is 50 857.3 to 1 decimal place.

3 The second and fifth terms of a geometric series are 75 and 129.6 respectively.

a Show that the first term of the series is 62.5

b Find the value of the tenth term of the series to 1 decimal place.

c Find the sum of the first 12 terms of the series to 1 decimal place.

4 **a** Prove that the sum, S_n, of the first n terms of a geometric series with first term a and common ratio r is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

b A geometric series has first term 2 and common ratio $\sqrt{2}$.

Given that the sum of the first n terms of the series is $126(\sqrt{2} + 1)$, find the value of n.

5 The first term of a geometric series is 18 and the sum to infinity of the series is 15.

a Find the common ratio of the series.

b Find the third term of the series.

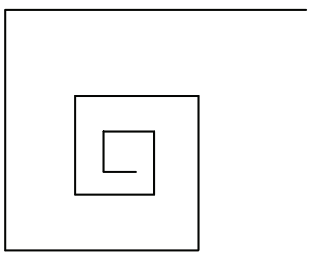
c Find the exact difference between the sum of the first eight terms of the series and the sum to infinity of the series.

6 The sum of the first n terms of a geometric series is given by $5(3^n - 1)$.

a Show that the third term of the series is 90.

b Find an expression for the nth term of the series in the form $k(3^n)$ where k is an exact fraction.

7



A student programs a computer to draw a series of straight lines with each line beginning at the end of the previous one and at right angles to it. The first line is 4 mm long and thereafter each line is 25% longer than the previous one, so that a spiral is formed as shown above.

a Find the length, in mm, of the eighth straight line drawn by the program.

b Find the total length of the spiral, in metres, when 20 straight lines have been drawn.

8 The second and fourth terms of a geometric series are 30 and 2.7 respectively.

Given that the common ratio, r, of the series is positive,

a find the value of r and the first term of the series,

b find the sum to infinity of the series.

9 **a** Evaluate $\sum_{r=3}^{10} 3^r$.

b Show that $\sum_{r=1}^{15} (2^r - 12r) = 64\,094$.

10 A geometric series has common ratio r and the nth term of the series is denoted by u_n.

Given that $u_1 = 64$ and that $u_3 - u_2 = 20$,

a show that $16r^2 - 16r - 5 = 0$,

b find the two possible values of r,

c find the fourth term of the series corresponding to each possible value of r.

d Taking the value of r such that the series converges, find the sum to infinity of the series.

11 A geometric series has first term 4 and common ratio $\frac{1}{2}$.

a Find the eighth term of the series as an exact fraction.

b Find the nth term of the series in the form 2^y where y is a function of n.

c Show that the sum of the first n terms of the series is $8 - 2^{3-n}$.

12 The sequence of terms $u_1, u_2, u_3, \ldots$ is defined by

$$u_n = 4 \times 3^n, \quad n \geq 1.$$

a Find u_6.

b Find the smallest value of t such that the sum of the first t terms of the sequence is greater than 10^{25}.

13 The sum of the first and third terms of a geometric series is 150. The sum of the second and fourth terms of the series is -75.

a Find the first term and common ratio of the series.

b Find the sum to infinity of the series.

14 Three consecutive terms of an arithmetic series are a, b and $(3a + 4)$ respectively.

a Find an expression for b in terms of a.

Given also that a, b and $(6a + 1)$ respectively are consecutive terms of a geometric series and that a and b are integers,

b find the values of a and b.

15 When a ball is dropped onto a horizontal floor it bounces such that it reaches a maximum height of 60% of the height from which it was dropped.

a Find the maximum height the ball reaches after its fourth bounce when it is initially dropped from 3 metres above the floor.

b Show that when the ball is dropped from a height of h metres above the floor it travels a total distance of $4h$ metres before coming to rest.

1 Expand each of the following, simplifying the coefficient in each term.

a $(1 + x)^4$ **b** $(1 - x)^5$ **c** $(1 + 4x)^3$ **d** $(1 - 2y)^3$

e $(1 + \frac{1}{2}x)^4$ **f** $(1 + \frac{1}{3}y)^3$ **g** $(1 + x^2)^5$ **h** $(1 - \frac{3}{2}x)^4$

2 Expand each of the following, simplifying the coefficient in each term.

a $(x + y)^3$ **b** $(a - b)^5$ **c** $(x + 2y)^4$ **d** $(2 + y)^3$

e $(3 - x)^3$ **f** $(5 + 2x)^4$ **g** $(3 - 4y)^5$ **h** $(3 + \frac{1}{2}x)^4$

3 Find the first four terms in the expansion in ascending powers of x of

a $(1 + x)^{10}$ **b** $(1 - x)^6$ **c** $(1 + 2x)^8$ **d** $(1 - \frac{1}{2}x)^7$

e $(1 + x^3)^6$ **f** $(2 + x)^9$ **g** $(3 - x)^7$ **h** $(2 + 5x)^{10}$

4 Find the coefficient indicated in the following expansions.

a $(1 + x)^{20}$, coefficient of x^3 **b** $(1 - x)^{14}$, coefficient of x^4

c $(1 + 4x)^9$, coefficient of x^2 **d** $(1 - 3y)^{14}$, coefficient of y^3

e $(1 - \frac{1}{3}x)^{12}$, coefficient of x^4 **f** $(1 - \frac{1}{2}x)^{16}$, coefficient of x^5

g $(1 + \frac{2}{5}x)^{15}$, coefficient of x^2 **h** $(1 + y^2)^8$, coefficient of y^6

5 Express each of the following in the required form where a and b are integers.

a $(1 + \sqrt{5})^3$ in the form $a + b\sqrt{5}$ **b** $(1 - \sqrt{3})^4$ in the form $a + b\sqrt{3}$

c $(2 + \sqrt{2})^3$ in the form $a + b\sqrt{2}$ **d** $(1 + 2\sqrt{3})^4$ in the form $a + b\sqrt{3}$

6 **a** Expand $(1 + x)^6$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.02^6 **ii** 0.99^6

giving your answers to 4 decimal places.

7 **a** Expand $(1 + 2y)^8$ in ascending powers of y up to and including the term in y^3, simplifying each coefficient.

b By substituting a suitable value of y into your answer for part **a**, obtain an estimate for

i 0.98^8 **ii** 1.01^8

giving your answers to 4 decimal places.

8 Expand and simplify

a $(1 + x)^4 + (1 - x)^4$ **b** $(1 - \frac{1}{3}x)^3 - (1 + \frac{1}{3}x)^3$

9 The coefficient of x^2 in the expansion of $(1 + ax)^4$ in ascending powers of x is 24, where a is a constant and $a < 0$. Find

a the value of a,

b the value of the coefficient of x^3 in the expansion.

1 Expand

a $(1 + 3x)^4$ **b** $(2 - x)^5$ **c** $(3 + 10x^2)^3$ **d** $(a + 2b)^5$

e $(x^2 - y)^3$ **f** $(5 + \frac{1}{2}x)^4$ **g** $(x + \frac{1}{x})^4$ **h** $(t - \frac{2}{t^2})^3$

2 Find the first four terms in the expansion in ascending powers of x of

a $(1 + 3x)^6$ **b** $(1 - \frac{1}{4}x)^8$ **c** $(5 - x)^7$ **d** $(3 + 2x^2)^{10}$

3 Find the coefficient indicated in the following expansions

a $(1 + x)^{15}$, coefficient of x^3 **b** $(1 - 2x)^{12}$, coefficient of x^4

c $(3 + x)^7$, coefficient of x^2 **d** $(2 - y)^{10}$, coefficient of y^5

e $(2 + t^3)^8$, coefficient of t^{15} **f** $(1 - \frac{1}{x})^9$, coefficient of x^{-3}

4 **a** Express $(\sqrt{2} - \sqrt{5})^4$ in the form $a + b\sqrt{10}$, where $a, b \in \mathbb{Z}$.

b Express $(\sqrt{2} + \frac{1}{\sqrt{3}})^3$ in the form $a\sqrt{2} + b\sqrt{3}$, where $a, b \in \mathbb{Q}$.

c Express $(1 + \sqrt{5})^3 - (1 - \sqrt{5})^3$ in the form $a\sqrt{5}$, where $a \in \mathbb{Z}$.

5 **a** Expand $(1 + \frac{x}{2})^{10}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 1.005^{10} **ii** 0.996^{10}

giving your answers to 5 decimal places.

6 **a** Expand $(3 + x)^8$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient.

b By substituting a suitable value of x into your answer for part **a**, obtain an estimate for

i 3.001^8 **ii** 2.995^8

giving your answers to 7 significant figures.

7 Expand and simplify

a $(1 + 10x)^4 + (1 - 10x)^4$ **b** $(2 - \frac{1}{3}x)^3 - (2 + \frac{1}{3}x)^3$

c $(1 + 4y)(1 + y)^3$ **d** $(1 - x)(1 + \frac{1}{x})^3$

8 Expand each of the following in ascending powers of x up to and including the term in x^3.

a $(1 + x^2)(1 - 3x)^{10}$ **b** $(1 - 2x)(1 + x)^8$

c $(1 + x + x^2)(1 - x)^6$ **d** $(1 + 3x - x^2)(1 + 2x)^7$

9 Find the term independent of y in each of the following expansions.

a $(y + \frac{1}{y})^8$ **b** $(2y - \frac{1}{2y})^{12}$ **c** $(\frac{1}{y} + y^2)^6$ **d** $(3y - \frac{1}{y^2})^9$

10 The coefficient of x^2 in the binomial expansion of $(1 + \frac{2}{5}x)^n$, where n is a positive integer, is 1.6

a Find the value of n.

b Use your value of n to find the coefficient of x^4 in the expansion.

11 Given that $y_1 = (1 - 2x)(1 + x)^{10}$ and $y_2 = ax^2 + bx + c$ and that when x is small, y_2 can be used as an approximation for y_1,

a find the values of the constants a, b and c,

b find the percentage error in using y_2 as an approximation for y_1 when $x = 0.2$

12 In the binomial expansion of $(1 + px)^q$, where p and q are constants and q is a positive integer, the coefficient of x is -12 and the coefficient of x^2 is 60.

Find

a the value of p and the value of q,

b the value of the coefficient of x^3 in the expansion.

13 **a** Expand $(3 - \frac{x}{3})^{12}$ as a binomial series in ascending powers of x up to and including the term in x^3, giving each coefficient as an integer.

b Use your series expansion with a suitable value of x to obtain an estimate for 2.998^{12}, giving your answer to 2 decimal places.

14 **a** Expand $(1 - x)^5$ as a binomial series in ascending powers of x.

b Express $(\sqrt{3} + 1)(\sqrt{3} - 2)$ in the form $A + B\sqrt{3}$, where $A, B \in \mathbb{Z}$.

c Hence express each of the following in the form $C + D\sqrt{3}$, where $C, D \in \mathbb{Z}$.

i $(\sqrt{3} + 1)^5(\sqrt{3} - 2)^5$

ii $(\sqrt{3} + 1)^6(\sqrt{3} - 2)^5$

15 **a** Expand $(1 + \frac{x}{2})^9$ in ascending powers of x up to and including the term in x^4.

Hence, or otherwise, find

b the coefficient of x^3 in the expansion of $(1 + \frac{x}{2})^9 - (1 - \frac{x}{2})^9$,

c the coefficient of x^4 in the expansion of $(1 + 2x)(1 + \frac{x}{2})^9$.

16 The term independent of x in the expansion of $(x^3 + \frac{a}{x^2})^5$ is -80.

Find the value of the constant a.

17 In the binomial expansion of $(1 + \frac{x}{k})^n$, where k is a non-zero constant, n is an integer and $n > 1$, the coefficient of x^2 is three times the coefficient of x^3.

a Show that $k = n - 2$.

Given also that $n = 7$,

b expand $(1 + \frac{x}{k})^n$ in ascending powers of x up to and including the term in x^4, giving each coefficient as a fraction in its simplest form.