

1 Express each of the following in the form $\log_a b = c$.

a $10^3 = 1000$

b $3^4 = 81$

c $256 = 2^8$

d $7^0 = 1$

e $3^{-3} = \frac{1}{27}$

f $32^{-\frac{1}{5}} = \frac{1}{2}$

g $19^1 = 19$

h $216 = 36^{\frac{3}{2}}$

2 Express each of the following using index notation.

a $\log_5 125 = 3$

b $\log_2 16 = 4$

c $5 = \log_{10} 100\,000$

d $\log_{23} 1 = 0$

e $\frac{1}{2} = \log_9 3$

f $\lg 0.01 = -2$

g $\log_2 \frac{1}{8} = -3$

h $\log_6 6 = 1$

3 Without using a calculator, find the exact value of

a $\log_7 49$

b $\log_4 64$

c $\log_2 128$

d $\log_3 27$

e $\log_5 625$

f $\log_8 8$

g $\log_7 1$

h $\log_{15} \frac{1}{15}$

i $\log_3 \frac{1}{9}$

j $\lg 0.001$

k $\log_{16} 2$

l $\log_4 8$

m $\log_9 243$

n $\log_{100} 0.001$

o $\log_{25} 125$

p $\log_{27} \frac{1}{9}$

4 Without using a calculator, find the exact value of x in each case.

a $\log_5 25 = x$

b $\log_2 x = 6$

c $\log_x 64 = 3$

d $\lg x = -3$

e $\log_x 16 = \frac{2}{3}$

f $\log_5 1 = x$

g $\log_x 9 = 1$

h $\lg 10^{12} = x$

i $2 \log_x 7 = 1$

j $\log_4 x = 1.5$

k $\log_x 0.1 = -\frac{1}{3}$

l $3 \log_8 x + 1 = 0$

5 Express in the form $\log_a n$

a $\log_a 4 + \log_a 7$

b $\log_a 10 - \log_a 5$

c $2 \log_a 6$

d $\log_a 9 - \log_a \frac{1}{3}$

e $\frac{1}{2} \log_a 25 + 2 \log_a 3$

f $\log_a 48 - 3 \log_a 2 - \frac{1}{2} \log_a 9$

6 Express in the form $p \log_q x$

a $\log_q x^5$

b $\frac{1}{2} \log_q x^{15}$

c $\log_q \frac{1}{x}$

d $\log_q \sqrt[3]{x}$

e $4 \log_q \frac{1}{\sqrt{x}}$

f $\log_q x^2 + \log_q x^5$

g $\log_q \frac{1}{x^2} + \log_q \frac{1}{x^3}$

h $3 \log_q x^2 - \frac{1}{2} \log_q x^4$

7 Express in the form $\lg n$

a $\lg 5 + \lg 4$

b $\lg 12 - \lg 6$

c $3 \lg 2$

d $4 \lg 3 - \lg 9$

e $\frac{1}{2} \lg 16 - \frac{1}{5} \lg 32$

f $1 + \lg 11$

g $\lg \frac{1}{50} + 2$

h $3 - \lg 40$

8 Without using a calculator, evaluate

a $\log_3 54 - \log_3 2$

b $\log_5 20 + \log_5 1.25$

c $\log_2 16 + \log_3 27$

d $\log_6 24 + \log_6 9$

e $\log_3 12 - \log_3 4$

f $\log_4 18 - \log_4 9$

g $\log_9 4 + \log_9 0.25$

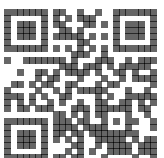
h $2 \lg 2 + \lg 25$

i $\frac{1}{3} \log_3 8 - \log_3 18$

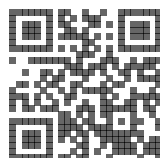
j $\frac{1}{3} \log_4 64 + 2 \log_5 25$

k $\frac{1}{2} \log_5 (1\frac{9}{16}) + 2 \log_5 10$

l $\log_3 5 - 2 \log_3 6 - \log_3 (3\frac{3}{4})$

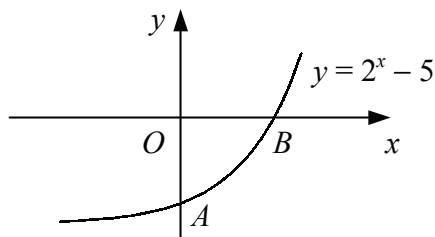


- 1 Express in the form $p \log_{10} a + q \log_{10} b$
- a $\log_{10} ab$ b $\log_{10} ab^7$ c $\log_{10} \frac{a^3}{b}$ d $\log_{10} a\sqrt{b}$
- e $\log_{10} (ab)^2$ f $\log_{10} \frac{1}{ab}$ g $\log_{10} \sqrt{a^3b^5}$ h $3 \log_{10} \frac{a^2}{\sqrt[3]{b}}$
- 2 Given that $y = \log_q 8$, express each of the following in terms of y .
- a $\log_q 64$ b $\log_q 2$ c $\log_q \frac{16}{q}$ d $\log_q 4q^3$
- 3 Given that $a = \lg 2$ and $b = \lg 3$, express each of the following in terms of a and b .
- a $\lg 18$ b $\lg 96$ c $\lg \frac{9}{16}$ d $\lg 6 - \lg 8$
- e $\lg \sqrt{6}$ f $\frac{3}{2} \lg 16 + \frac{1}{2} \lg 81$ g $4 \lg 3 - 3 \lg 6$ h $\lg 60 + \lg 20 - 2$
- 4 Without using a calculator, evaluate
- a $\frac{1}{3} \log_5 1000 - \frac{1}{2} \log_5 4$ b $2 \log_{12} 4 + \frac{1}{2} \log_{12} 81$ c $\log_4 12 + \log_4 \frac{2}{3}$
- d $\frac{\log_7 81}{\log_7 3}$ e $3 \log_{27} 12 - 2 \log_{27} 72$ f $\frac{\log_{11} 25}{\log_{11} \frac{1}{5}}$
- 5 Solve each equation, giving your answers correct to 3 significant figures.
- a $\log_3 x = 1.8$ b $\log_5 x = -0.3$ c $\log_8 (x - 3) = 2.1$
- d $\log_4 (\frac{1}{2}x + 1) = 3.2$ e $15 - \log_2 3y = 9.7$ f $\log_6 (1 - 5t) + 4.2 = 3.6$
- 6 Express in the form $\log_2 [f(x)]$
- a $5 \log_2 x$ b $\log_2 x + \log_2 (x + 4)$ c $2 \log_2 x + \frac{1}{5} \log_2 x^5$
- d $3 \log_2 (x - 2) - 4 \log_2 x$ e $\log_2 (x^2 - 1) - \log_2 (x + 1)$ f $\log_2 x - \frac{1}{2} \log_2 x^4 + \frac{1}{3} \log_2 x^2$
- 7 Solve each of the following equations.
- a $\log_3 x + \log_3 5 = \log_3 (2x + 3)$ b $\log_9 x + \log_9 10 = \frac{3}{2}$
- c $\log_4 x - \log_4 (x - 1) = \log_4 3 + \frac{1}{2}$ d $\log_5 5x - \log_5 (x + 2) = \log_5 (x + 6) - \log_5 x$
- e $2 \log_6 x = \log_6 (2x - 5) + \log_6 5$ f $\log_7 4x = \log_7 \frac{1}{x-6} + 1$
- 8 Solve each pair of simultaneous equations.
- a $\log_x y = 2$ b $\log_5 x - 2 \log_5 y = \log_5 2$
 $xy = 27$ $x + y^2 = 12$
- c $\log_2 x = 3 - 2 \log_2 y$ d $\log_y x = \frac{3}{2}$
 $\log_y 32 = -\frac{5}{2}$ $x^{\frac{1}{3}} + 3y^{\frac{1}{2}} = 20$
- e $\log_a x + \log_a 3 = \frac{1}{2} \log_a y$ f $\log_{10} y + 2 \log_{10} x = 3$
 $3x + y = 20$ $\log_2 y - \log_2 x = 3$



- 1 Find, to 3 significant figures, the value of
 a $\log_{10} 60$ b $\log_{10} 6$ c $\log_{10} 253$ d $\log_{10} 0.4$
- 2 Solve each equation, giving your answers to 2 decimal places.
 a $10^x = 14$ b $2(10^x) - 8 = 0$ c $10^{3x} = 49$
 d $10^{x-4} = 23$ e $10^{2x+1} = 130$ f $100^x - 5 = 0$
- 3 Show that $\log_a b = \frac{\log_c b}{\log_c a}$, where a , b and c are positive constants.
- 4 Find, to 3 significant figures, the value of
 a $\log_2 7$ b $\log_{20} 172$ c $\log_5 49$ d $\log_9 4$
- 5 Solve each equation, giving your answers to 3 significant figures.
 a $3^x = 12$ b $2^x = 0.7$ c $8^{-y} = 3$ d $4^{\frac{1}{2}x} - 0.3 = 0$
 e $5^{t+3} = 24$ f $16 - 3^{4+x} = 0$ g $7^{2x+4} = 12$ h $5(2^{3x+1}) = 62$
 i $4^{2-3x} = 32.7$ j $5^x = 6^{x-1}$ k $7^{y+2} = 9^{y+1}$ l $4^{5-x} = 11^{2x-1}$
 m $4^{\frac{1}{2}x+3} - 5^{1-2x} = 0$ n $2^{3y-2} = 3^{2y+5}$ o $7^{2x+5} = 7(11^{3x-4})$ p $3^{2x} = 3^{x-1} \times 2^{4+x}$
- 6 Solve the following equations, giving your answers to 2 decimal places where appropriate.
 a $2^{2x} + 2^x - 6 = 0$ b $3^{2x} - 5(3^x) + 4 = 0$ c $5^{2x} + 12 = 8(5^x)$
 d $2(4^x) + 3(4^{-x}) = 7$ e $2^{2y+1} + 7(2^y) - 15 = 0$ f $3^{2x+1} - 17(3^x) + 10 = 0$
 g $25^t + 5^{t+1} - 24 = 0$ h $3^{2x+1} + 15 = 2(3^{x+2})$ i $3(16^x) - 4^{x+2} + 5 = 0$
- 7 Sketch each pair of curves on the same diagram, showing the coordinates of any points of intersection with the coordinate axes.
 a $y = 2^x$ b $y = 3^x$ c $y = 4^x$ d $y = 2^x$
 $y = 5^x$ $y = (\frac{1}{3})^x$ $y = 4^x - 1$ $y = 2^{x+3}$
- 8 A curve has the equation $y = 2 + a^x$ where a is a constant and $a > 1$.
 a Sketch the curve, showing the coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes.
 Given also that the curve passes through the point (3, 29),
 b find the value of a .

9



The diagram shows the curve with equation $y = 2^x - 5$ which intersects the coordinate axes at the points A and B . Find the length AB correct to 3 significant figures.

