1 Write down an equation of the circle with the given centre and radius in each case．
a centre $(0,0)$
radius 5
b centre $(1,3)$
radius 2
c centre $(4,-6)$
radius 1
d centre $(-1,-8)$
radius 3
e centre $\left(-\frac{1}{2}, \frac{1}{2}\right)$
radius $\frac{1}{2}$
f centre $(-3,9)$
radius $2 \sqrt{3}$

2 Write down the coordinates of the centre and the radius of each of the following circles．
a $x^{2}+y^{2}=16$
b $(x-6)^{2}+(y-1)^{2}=81$
c $(x+1)^{2}+(y-4)^{2}=121$
d $(x-7)^{2}+y^{2}=0.09$
e $(x+2)^{2}+(y+5)^{2}=32$
f $(x-8)^{2}+(y+9)^{2}=108$

3 Find the coordinates of the centre and the radius of each of the following circles．
a $x^{2}+y^{2}-4 y+3=0$
b $x^{2}+y^{2}-2 x-10 y-23=0$
c $x^{2}+y^{2}+12 x-8 y+36=0$
d $x^{2}+y^{2}-2 x+16 y=35$
e $x^{2}+y^{2}=8 x-6 y$
f $x^{2}+y^{2}+10 x-2 y-19=0$
g $4 x^{2}+4 y^{2}-4 x-24 y+1=0$
h $9 x^{2}+9 y^{2}+6 x-24 y+8=0$

4 Find an equation of the circle
a with centre $(1,-2)$ which passes through the point $(4,2)$ ，
b with centre $(-5,7)$ which passes through the point $(0,5)$ ．
5 Find an equation of the circle in which $A B$ is a diameter in each case．
a $A(1,-2) \quad B(3,-2)$
b $A(-7,2)$
B $(1,8)$
c $A(1,1)$
$B(4,0)$

6 The points $P(0,1), Q(3,10)$ and $R(6,9)$ all lie on circle $C$ ．
a Show that $\angle P Q R$ is a right－angle．
b Hence，show that $C$ has the equation $x^{2}+y^{2}-6 x-10 y+9=0$ ．
7 Find in each case whether the given point lies inside，outside or on the given circle．
a $(0,-9)$
$x^{2}+y^{2}=64$
b $(4,7)$
$x^{2}+y^{2}-2 x-6 y-26=0$
c $(7,-3)$
$x^{2}+y^{2}+10 x-4 y=140$
d $(-4,1)$
$x^{2}+y^{2}+2 x+8 y-13=0$

8 The point $P$ lies on the circle with equation $x^{2}+y^{2}+12 x-6 y+27=0$ and the point $Q$ has coordinates $(8,1)$ ．Find the minimum length of $P Q$ giving your answer in the form $k \sqrt{2}$ ．

9 Find an equation of the circle which crosses the $x$－axis at the points $(2,0)$ and $(8,0)$ and touches the $y$－axis at the point $(0,4)$ ．

10 Given that the circle with equation $x^{2}+y^{2}+8 x-12 y+k=0$ does not touch or cross either of the coordinate axes，find the set of possible values of the constant $k$ ．

11 The circle $C$ passes through the points $P, Q$ and $R$ with coordinates $(-2,-2),(2,-4)$ and $(7,1)$ respectively．
a Find an equation of the perpendicular bisector of the points $P$ and $Q$ ．
b Find the coordinates of the centre of $C$ ．
c Find an equation of $C$ ．

12 The circle $C$ has the equation $x^{2}+y^{2}-4 x-4 y-28=0$ ．
a Find the distance of the point $A(10,8)$ from the centre of $C$ ．
The tangent to $C$ at the point $B$ passes through $A$ ．
b Find the length $A B$ ．
13 A circle has the equation $x^{2}+y^{2}+6 x-2 y=0$ and passes through the point $P$ ．
Given that the tangent to the circle at $P$ passes through the point $Q(2,6)$ ，find the exact length $P Q$ in its simplest form．

14 The circle $C$ has the equation $x^{2}+y^{2}-6 x-10 y+16=0$ and passes through the point $A(6,2)$ ．
a Find the coordinates of the centre of $C$ ．
b Find the gradient of the normal to the circle at $A$ ．
c Find an equation of the normal to the circle at $A$ ．
15 Find an equation of
a the normal to the circle with equation $x^{2}+y^{2}+4 x=13$ at the point $(-1,4)$ ，
b the tangent to the circle with equation $x^{2}+y^{2}+2 x+4 y-40=0$ at the point $(5,1)$ ，
c the tangent to the circle with equation $x^{2}+y^{2}-10 x+4 y+4=0$ at the point $(2,2)$ ．
16 Find the coordinates of the points where the circle with equation $x^{2}+y^{2}-6 x+6 y-16=0$ intersects the coordinate axes．

17 Find in each case the coordinates of the points where the line $l$ intersects the circle $C$ ．
a $l: y=x-4$
$C: x^{2}+y^{2}=10$
b $l: 3 x+y=17$
$C: x^{2}+y^{2}-4 x-2 y-15=0$
c $l: y=2 x+2$
$C: 4 x^{2}+4 y^{2}+4 x-8 y-15=0$

18 The line with equation $y=1-x$ intersects the circle with equation $x^{2}+y^{2}+6 x+2 y=27$ at the points $A$ and $B$ ．
Find the length of the chord $A B$ ，giving your answer in the form $k \sqrt{2}$ ．
19 Show that the line with equation $y=2 x+1$ is a tangent to the circle with equation $x^{2}+y^{2}-8 x-8 y+27=0$ and find the coordinates of the point where they touch．

20 The line with equation $y=x+k$ is a tangent to the circle with equation $x^{2}+y^{2}+6 x-8 y+17=1$ Find the two possible values of $k$ ．

21 The line with equation $y=m x$ is a tangent to the circle with equation $x^{2}+y^{2}-8 x-16 y+72=0$ Find the two possible values of $m$ ．

22 The line with equation $2 x+3 y=k$ is a tangent to the circle with equation $x^{2}+y^{2}+6 x+4 y=0$ ． Find the two possible values of $k$ ．

23 The circle with equation $x^{2}+y^{2}-4 x-6 y=7$ crosses the $y$－axis at the points $A$ and $B$ ．
a Find the coordinates of the points $A$ and $B$ ．
b Find the coordinates of the point where the tangent to the circle at $A$ intersects the tangent to the circle at $B$ ．

1 The circle $C$ has centre $(3,-2)$ and radius 5.
a Write down an equation of $C$ in cartesian form．
The line $y=2 x-3$ intersects $C$ at the points $A$ and $B$ ．
b Show that $A B=4 \sqrt{5}$ ．
2 The line $A B$ is a diameter of circle $C$ ．
Given that $A$ has coordinates $(-5,6)$ and $B$ has coordinates $(3,8)$ ，find
a the coordinates of the centre of $C$ ，
b a cartesian equation for $C$ ，
c an equation of the tangent to $C$ at $A$ ．
3 The circle $C$ has equation $x^{2}+y^{2}+8 x-16 y+62=0$ ．
a Find the coordinates of the centre of $C$ and the exact radius of $C$ ．
The line $l$ has equation $y=2 x+1$ ．
b Show that the minimum distance between $l$ and $C$ is $3(\sqrt{5}-\sqrt{2})$ ．
4


The diagram shows rectangle $P Q R S$ and circles $C_{1}$ and $C_{2}$ ．
Each circle touches the other circle and three sides of the rectangle．The coordinates of the corners of the rectangle are $P(0,4), Q(1,1), R(7,3)$ and $S(6,6)$ ．
a Find the radius of $C_{1}$ ．
b Find the coordinates of the point where the two circles touch．
c Show that $C_{1}$ has equation $2 x^{2}+2 y^{2}-8 x-12 y+21=0$ ．
$5 \quad$ The circle $C$ touches the $y$－axis at the point $A(0,3)$ and passes through the point $B(2,7)$ ．
a Find an equation of the perpendicular bisector of $A B$ ．
b Find an equation for $C$ ．
c Show that the tangent to $C$ at $B$ has equation

$$
3 x-4 y+22=0
$$

6 The point $P(x, y)$ moves such that its distance from the point $A(-3,4)$ is twice its distance from the point $B(0,-2)$ ．
Show that the locus of $P$ is a circle and find the coordinates of the centre and the exact radius of this circle．

7 The points $P(-4,9)$ and $Q(-2,-5)$ are such that $P Q$ is a diameter of circle $C$ ．
a Find the coordinates of the centre of $C$ ．
b Find an equation for $C$ ．
c Show that the point $R(2,7)$ lies on $C$ ．
d Hence，state the size of $\angle P R Q$ ，giving a reason for your answer．
8


The diagram shows circles $C_{1}$ and $C_{2}$ ，which both pass through the point $P$ ，and the common tangent to the circles at $P$ ，the line $l$ ．
Circle $C_{1}$ has the equation $x^{2}+y^{2}-4 y-16=0$ ．
a Find the coordinates of the centre of $C_{1}$ ．
Circle $C_{2}$ has the equation $x^{2}+y^{2}-2 x-8 y-60=0$ ．
b Find an equation of the straight line passing through the centre of $C_{1}$ and the centre of $C_{2}$ ．
c Find an equation of line $l$ ．
9 The circle $C$ has equation $x^{2}+y^{2}-8 x+4 y+12=0$ ．
a Find the coordinates of the centre of $C$ and the radius of $C$ ．
The point $P$ has coordinates $(3,5)$ and the point $Q$ lies on $C$ ．
b Find the largest and smallest values of the length $P Q$ ，giving your answers in the form $k \sqrt{2}$ ．
c Find the length of $P Q$ correct to 3 significant figures when the line $P Q$ is a tangent to $C$ ．
10


The diagram shows the circle $C$ and the line $y=x$ ．
Given that circle $C$ has centre $(a, b)$ ，where $a$ and $b$ are positive constants，and that $C$ touches the $x$－axis，
a find a cartesian equation for $C$ in terms of $a$ and $b$ ．
Given also that the line $y=x$ is a tangent to $C$ ，
b show that $a=(1+\sqrt{2}) b$ ．

