

# Solomon Practice Paper

## Pure Mathematics 6B

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

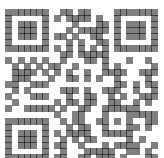
Question	Points	Score
1	5	
2	5	
3	5	
4	6	
5	11	
6	12	
7	14	
8	17	
Total:	75	

How I can achieve better:

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1. Given that  $x$  is so small that terms in  $x^3$  and higher powers of  $x$  may be neglected, find the values of the constants  $a$  and  $b$  for which [5]

$$\frac{\ln(1 + ax)}{1 + bx} = 3x + \frac{3}{2}x^2.$$

2. Given that

$$|z + 1 - 4i| = 1,$$

- (a) sketch, in an Argand diagram, the locus of  $z$ , [2]  
 (b) find the maximum value of  $\arg(z)$  in degrees to one decimal place. [3]

Total: 5

3. (a) Show that [2]

$$\cosh(ix) = \cos(x) \quad \text{where } x \in \mathbb{R}.$$

- (b) Hence, or otherwise, solve the equation [3]

$$\cosh(ix) = e^{ix}$$

for  $0 \leq x < 2\pi$ .

Total: 5

4. Given that [6]

$$u_{n+2} = 5u_{n+1} - 6u_n \quad n \geq 1, \quad u_1 = 2 \quad \text{and} \quad u_2 = 4,$$

prove by induction that  $u_n = 2^n$  for all integers  $n, n \geq 1$ .

- 5.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -4 \\ x & 3 & -1 \end{pmatrix}.$$

- (a) Given that  $\lambda = -1$  is an eigenvalue of  $\mathbf{M}$ , find the value of  $x$ . [3]  
 (b) Show that  $\lambda = -1$  is the only real eigenvalue of  $\mathbf{M}$ . [6]  
 (c) Find an eigenvector corresponding to the eigenvalue  $\lambda = -1$ . [2]

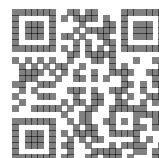
Total: 11

6. A student is looking at different methods of solving the differential equation

$$\frac{dy}{dx} = xy, \quad y = 1 \quad \text{when} \quad x = 0.2.$$

The first method the student tries is to use the approximation

$$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_0}{h}$$



twice with a step length of 0.1 to obtain an estimate for  $y$  at  $x = 0.4$ .

- (a) Find the value of the student's estimate for  $y$  at  $x = 0.4$ . [6]

The student then realises that the exact value of  $y$  at  $x = 0.4$  can be found using integration.

- (b) Use integration to find the exact value of  $y$  at  $x = 0.4$ . [4]

- (c) Find, correct to 1 decimal place, the percentage error in the estimated value in part (a). [2]

Total: 12

7. (a) Given that  $z = \cos(\theta) + \mathbf{i} \sin(\theta)$ , show that [3]

$$z^n + \frac{1}{z^n} = 2 \cos(n\theta) \quad \text{and} \quad z^n - \frac{1}{z^n} = 2\mathbf{i} \sin(n\theta),$$

where  $n$  is a positive integer.

- (b) Given that [8]

$$\cos^4(\theta) + \sin^4(\theta) = A \cos(4\theta) + B,$$

find the values of the constants  $A$  and  $B$ .

- (c) Hence find the exact value of [3]

$$\int_0^{\frac{\pi}{8}} \cos^4(\theta) + \sin^4(\theta) \, d\theta.$$

Total: 14

8. The points  $A, B, C$  and  $D$  have coordinates  $(3, -1, 2)$ ,  $(-2, 0, -1)$ ,  $(1, 2, 6)$  and  $(-1, -5, 8)$  respectively, relative to the origin  $O$ .

- (a) Find  $\overrightarrow{AB} \times \overrightarrow{AC}$ . [5]

- (b) Find the volume of the tetrahedron  $ABCD$ . [3]

The plane  $\Pi$  contains the points  $A, B$  and  $C$ .

- (c) Find a vector equation of  $\Pi$  in the form  $\mathbf{r} \cdot \mathbf{n} = p$ . [3]

The perpendicular from  $D$  to  $\Pi$  meets the plane at the point  $E$ .

- (d) Find the coordinates of  $E$ . [6]

Total: 17

