Solomon Practice Paper

Pure Mathematics 6B

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	5	
2	5	
3	5	
4	6	
5	11	
6	12	
7	14	
8	17	
Total:	75	

How I can achieve better:

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1. Given that x is so small that terms in x^3 and higher powers of x may be neglected, find the values of the constants a and b for which [5]

$$\frac{\ln(1+ax)}{1+bx} = 3x + \frac{3}{2}x^2.$$

2. Given that

$$|z+1-4\mathbf{i}| = 1,$$

   (a) sketch, in an Argand diagram, the locus of z, [2]

   (b) find the maximum value of $\arg(z)$ in degrees to one decimal place. [3]

Total: 5

3. (a) Show that [2]

$$\cosh(\mathbf{i}x) = \cos(x) \quad \text{where} \quad x \in \mathbb{R}.$$

   (b) Hence, or otherwise, solve the equation [3]

$$\cosh(\mathbf{i}x) = \mathrm{e}^{\mathbf{i}x}$$

   for $0 \le x < 2\pi$.

Total: 5

4. Given that [6]

$$u_{n+2} = 5u_{n+1} - 6u_n \qquad n \ge 1, \qquad u_1 = 2 \quad \text{and} \quad u_2 = 4,$$

   prove by induction that $u_n = 2^n$ for all integers $n, n \ge 1$.

5.

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -4 \\ x & 3 & -1 \end{pmatrix}.$$

   (a) Given that $\lambda = -1$ is an eigenvalue of $\mathbf{M}$, find the value of x. [3]

   (b) Show that $\lambda = -1$ is the only real eigenvalue of $\mathbf{M}$. [6]

   (c) Find an eigenvector corresponding to the eigenvalue $\lambda = -1$. [2]

Total: 11

6. A student is looking at different methods of solving the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = xy, \qquad y = 1 \quad \text{when} \quad x = 0.2.$$

   The first method the student tries is to use the approximation

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_0}{h}$$

twice with a step length of 0.1 to obtain an estimate for y at $x = 0.4$.

(a) Find the value of the student's estimate for y at $x = 0.4$. [6]

The student then realises that the exact value of y at $x = 0.4$ can be found using integration.

(b) Use integration to find the exact value of y at $x = 0.4$. [4]

(c) Find, correct to 1 decimal place, the percentage error in the estimated value in part (a). [2]

Total: 12

7. (a) Given that $z = \cos(\theta) + \mathbf{i}\sin(\theta)$, show that [3]

$$z^n + \frac{1}{z^n} = 2\cos(n\theta) \quad \text{and} \quad z^n - \frac{1}{z^n} = 2\mathbf{i}\sin(n\theta),$$

where n is a positive integer.

(b) Given that [8]

$$\cos^4(\theta) + \sin^4(\theta) = A\cos(4\theta) + B,$$

find the values of the constants A and B.

(c) Hence find the exact value of [3]

$$\int_0^{\frac{\pi}{8}} \cos^4(\theta) + \sin^4(\theta)\,\mathrm{d}\theta.$$

Total: 14

8. The points A, B, C and D have coordinates $(3, -1, 2)$, $(-2, 0, -1)$, $(1, 2, 6)$ and $(-1, -5, 8)$ respectively, relative to the origin O.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$. [5]

(b) Find the volume of the tetrahedron $ABCD$. [3]

The plane Π contains the points A, B and C.

(c) Find a vector equation of Π in the form $\mathbf{r}.\mathbf{n} = p$. [3]

The perpendicular from D to Π meets the plane at the point E.

(d) Find the coordinates of E. [6]

Total: 17