

Solomon Practice Paper

Pure Mathematics 5G

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

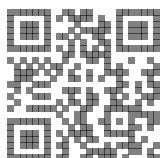
Question	Points	Score
1	7	
2	7	
3	8	
4	9	
5	12	
6	14	
7	18	
Total:	75	

How I can achieve better:

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1. Given that $y = e^{\arctan(x)}$,

(a) find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

The curve $y = e^{\arctan(x)}$ has a point of inflexion.

(b) Find the coordinates of this point of inflexion. [3]

Total: 7

2. (a) Prove that

$$\frac{d}{dx} \operatorname{arcosh}(x) = \frac{1}{\sqrt{x^2 - 1}}.$$

(b) Find [4]

$$\int \operatorname{arcosh}(x) dx.$$

Total: 7

3. Find

$$\int_0^{\frac{\pi}{4}} \frac{1}{1 + \sin(2x)} dx.$$

[8]

4. (a) Find

$$\int \frac{1}{\sqrt{4x^2 - 4x + 10}} dx.$$

[6]

(b) Hence evaluate

$$\int_{\frac{1}{2}}^2 \frac{1}{\sqrt{4x^2 - 4x + 10}} dx.$$

[3]

giving your answer in terms of natural logarithms.

Total: 9

5. (a) On the same axes sketch the curves with equations $y = 2 - \tanh(x)$ and $y = 3 \operatorname{sech}(x)$, giving the coordinates of the points of intersection of the curves with the coordinate axes and the equations of the asymptotes. [5]

(b) Solve the equation [7]

$$2 - \tanh(x) = 3 \operatorname{sech}(x),$$

giving your answers to 2 decimal places.

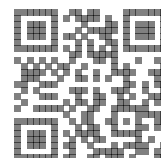
Total: 12

6.

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n(x) dx, \quad n \geq 0.$$

(a) Show that [7]

$$I_n = \frac{n-1}{n} I_{n-2}, \quad n \geq 2.$$



The curve C is defined by $y = \sin^2(x), 0 \leq x \leq \pi$.

The area bounded by C and the positive x -axis is rotated through 2π radians about the x -axis.

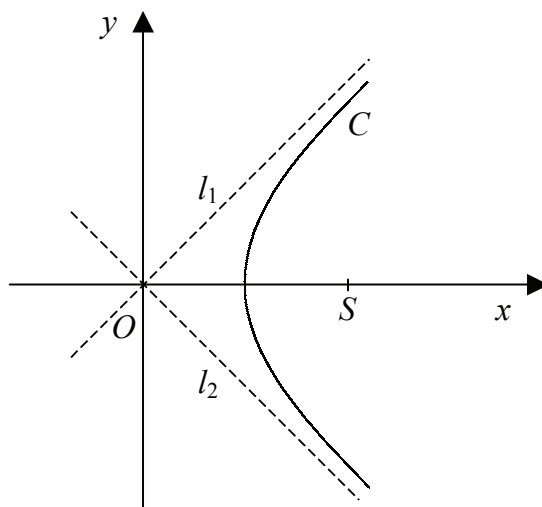
(b) Find the volume of the solid generated giving your answer in terms of π . [7]

Total: 14

7. Figure shows the curve C which is part of the hyperbola with parametric equations

$$x = a \cosh(t), \quad \text{and} \quad y = 2a \sinh(t),$$

where a is a positive constant and $x \geq a$.



The lines l_1 and l_2 are asymptotes to C .

(a) Show that the radius of curvature of C at its vertex is $4a$. [6]

(b) Show that an equation of the tangent to C at the point $P(\cosh(p), 2a \sinh(p))$ is [4]

$$2x \cosh(p) - y \sinh(p) = 2a.$$

The tangent to the curve C at P meets the asymptote l_1 at Q .

Given that QS is parallel to the y -axis, where S is the focus,

(c) show that $p = \frac{1}{2} \ln(5)$. [8]

Total: 18

