Solomon Practice Paper

Pure Mathematics 5F

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	4	
2	7	
3	7	
4	9	
5	10	
6	11	
7	12	
8	15	
Total:	75	

How I can achieve better:

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1. [4]

$$\mathrm{f}(x) = \operatorname{arctanh}(\sin(x)).$$

Show that $\mathrm{f}'(x) = \sec(x)$.

2. Find the length of the arc of the curve with equation $y = \ln(\sec(x))$ between $x = 0$ and $x = \frac{\pi}{3}$, giving your answer in terms of natural logarithms. [7]

3. A curve has parametric equations [7]

$$x = t^2, \quad \text{and} \quad y = t^3.$$

Show that the radius of curvature of the curve at the point $(1, 1)$ is $\frac{13\sqrt{13}}{6}$.

4.

$$I_n = \int_1^{\mathrm{e}} (\ln(x))^n \, \mathrm{d}x.$$

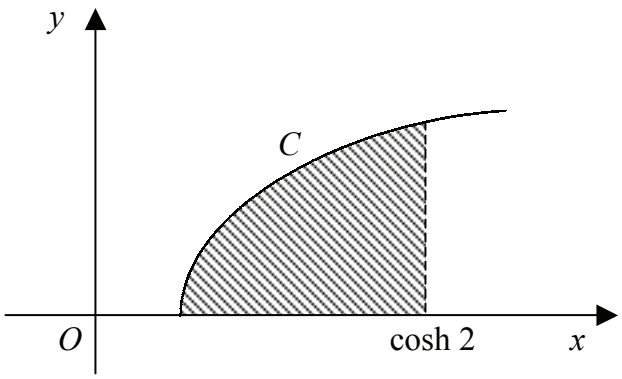
(a) Prove that, for $n \in \mathbb{Z}^+$, [4]

$$I_n = \mathrm{e} - nI_{n-1}.$$

(b) Find I_3, leaving your answer in terms of e. [5]

Total: 9

5. Figure shows the curve C which has equation $y = \operatorname{arcosh}(x)$. [10]



The shaded region bounded by C, the x-axis and the line $x = \cosh(2)$ is rotated through 2π about the y-axis.

The volume of revolution of the solid generated is $a\pi$.

Find the value of a to one decimal place.

6.

$$\mathrm{f}(x) \equiv \frac{3x - 7}{(x + 1)(x^2 + 4)}, \qquad x \neq -1.$$

(a) Express $\mathrm{f}(x)$ in partial fractions. [4]

(b) Show that [7]

$$\int_0^2 \mathrm{f}(x)\,\mathrm{d}x = \frac{\pi}{8} + \ln\left(\frac{2}{9}\right).$$

Total: 11

7. The ellipse C has equation $\dfrac{x^2}{a} + \dfrac{y^2}{b} = 1$, where a and b are positive constants and $a > b$.

(a) Find an equation of the normal to C at the point $P(a\cos(\theta), b\sin(\theta))$. [5]

The normal to C at P meets the x-axis at Q.

R is the foot of the perpendicular from P to the x-axis.

(b) Show that $\dfrac{OQ}{OR} = \mathrm{e}^2$, where e is the eccentricity of C. [7]

Total: 12

8. (a) Using the definitions of hyperbolic functions in terms of exponential functions prove that [6]

$$\operatorname{arcsinh}(x) = \ln\left(x + \sqrt{x^2+1}\right)$$

(b) On the same axes sketch the graphs of $y = \sinh(x)$ and $y = \operatorname{arcsinh}(x)$. [3]

(c) Solve the equation [6]

$$x = \sinh\left[\ln(3x-2)\right], \qquad x > \frac{2}{3}.$$

Total: 15