Solomon Practice Paper

Pure Mathematics 5E

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	4	
2	6	
3	9	
4	9	
5	11	
6	11	
7	11	
8	14	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. A student without a calculator must find the value of x given that $\operatorname{arctanh}(x) = \ln(3)$. With clear working, show how the student could find x and state the value he should obtain. [4]

2. $$\mathrm{f}(x) = \sin(2x) - x\cosh^2(x).$$

   (a) Find $\mathrm{f}'(x)$. [3]

   (b) Show that the curve with equation $y = \mathrm{f}(x)$ has a stationary point in the interval $0.3 < x < 0.4$. [3]

   Total: 6

3. Given that [9]
$$\int_0^{\frac{2\pi}{3}} \frac{1}{5 + 4\cos(x)}\,\mathrm{d}x = a\pi, \qquad a \in \mathbb{Q},$$
use the substitution $t = \tan\left(\frac{1}{2}x\right)$ to find the value of a.

4. The curve C has equation [9]
$$y = a\cosh\left(\frac{x}{a}\right),$$
where a is a positive constant.

   The area bounded by the curve C, the x-axis and the lines $x = -a$ and $x = a$ is rotated through 2π radians about the x-axis.

   Show that the curved surface area of the solid generated is $\pi a^2(\sinh(2) + 2)$.

5. The intrinsic equation of the curve C is $s = 2\psi$.

   Given that s is measured from the origin,

   (a) find a Cartesian equation of C, [9]

   (b) sketch C. [2]

   Total: 11

6. (a) Using the definitions of hyperbolic functions in terms of exponential functions, prove that [4]
$$\cosh(x + y) \equiv \cosh(x)\cosh(y) + \sinh(x)\sinh(y).$$

   Given that
$$5\cosh(x) + 4\sinh(x) \equiv R\cosh(x + \alpha),$$
find

   (b) the value of R, [3]

   (c) the value of α, giving your answer in terms of natural logarithms. [3]

   (d) Hence, or otherwise, state the minimum value of $5\cosh(x) + 4\sinh(x)$. [1]

   Total: 11

7.
$$I_n = \int_0^1 x^n e^{x^2}\,dx, \quad n \geq 0.$$

(a) Show that [5]

$$I_n = \frac{1}{2}e - \frac{1}{2}(n-1)I_{n-2}, \quad n \geq 2.$$

(b) Hence find [6]

$$I_n = \int_0^1 x^5 e^{x^2}\,dx,$$

giving your answer in terms of e.

Total: 11

8. The line with equation $y = mx + c$ is a tangent to the parabola with equation $y^2 = 8x$.

(a) Show that $mc = 2$. [5]

The lines l_1 and l_2 are tangents to both the parabola with equation $y^2 = 8x$ and the circle with equation $x^2 + y^2 = 2$.

(b) Find the equations of l_1 and l_2. [9]

Total: 14