

# Solomon Practice Paper

## Pure Mathematics 5E

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

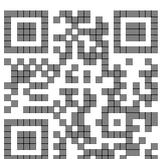
Question	Points	Score
1	4	
2	6	
3	9	
4	9	
5	11	
6	11	
7	11	
8	14	
Total:	75	

How I can achieve better:

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1. A student without a calculator must find the value of  $x$  given that  $\operatorname{arctanh}(x) = \ln(3)$ . With clear working, show how the student could find  $x$  and state the value he should obtain. [4]

2.

$$f(x) = \sin(2x) - x \cosh^2(x).$$

- (a) Find  $f'(x)$ . [3]
- (b) Show that the curve with equation  $y = f(x)$  has a stationary point in the interval  $0.3 < x < 0.4$ . [3]

Total: 6

3. Given that [9]

$$\int_0^{\frac{2\pi}{3}} \frac{1}{5 + 4 \cos(x)} dx = a\pi, \quad a \in \mathbb{Q},$$

use the substitution  $t = \tan\left(\frac{1}{2}x\right)$  to find the value of  $a$ .

4. The curve  $C$  has equation [9]

$$y = a \cosh\left(\frac{x}{a}\right),$$

where  $a$  is a positive constant.

The area bounded by the curve  $C$ , the  $x$ -axis and the lines  $x = -a$  and  $x = a$  is rotated through  $2\pi$  radians about the  $x$ -axis.

Show that the curved surface area of the solid generated is  $\pi a^2(\sinh(2) + 2)$ .

5. The intrinsic equation of the curve  $C$  is  $s = 2\psi$ .

Given that  $s$  is measured from the origin,

- (a) find a Cartesian equation of  $C$ , [9]
- (b) sketch  $C$ . [2]

Total: 11

6. (a) Using the definitions of hyperbolic functions in terms of exponential functions, prove that [4]

$$\cosh(x + y) \equiv \cosh(x) \cosh(y) + \sinh(x) \sinh(y).$$

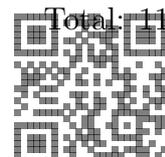
Given that

$$5 \cosh(x) + 4 \sinh(x) \equiv R \cosh(x + \alpha),$$

find

- (b) the value of  $R$ , [3]
- (c) the value of  $\alpha$ , giving your answer in terms of natural logarithms. [3]
- (d) Hence, or otherwise, state the minimum value of  $5 \cosh(x) + 4 \sinh(x)$ . [1]

Total: 14



7.

$$I_n = \int_0^1 x^n e^{x^2} dx, \quad n \geq 0.$$

(a) Show that

[5]

$$I_n = \frac{1}{2}e - \frac{1}{2}(n-1)I_{n-2}, \quad n \geq 2.$$

(b) Hence find

[6]

$$I_n = \int_0^1 x^5 e^{x^2} dx,$$

giving your answer in terms of e.

Total: 11

8. The line with equation  $y = mx + c$  is a tangent to the parabola with equation  $y^2 = 8x$ .(a) Show that  $mc = 2$ .

[5]

The lines  $l_1$  and  $l_2$  are tangents to both the parabola with equation  $y^2 = 8x$  and the circle with equation  $x^2 + y^2 = 2$ .

(b) Find the equations of  $l_1$  and  $l_2$ .

[9]

Total: 14

