Solomon Practice Paper

Pure Mathematics 5D

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	5	
2	5	
3	11	
4	12	
5	13	
6	13	
7	16	
Total:	75	

How I can achieve better:

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1.
$$y = \frac{\text{cosech}(x)}{x^2 + 1}.$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [4]

(b) Find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0.5$, giving your answer to 2 decimal places. [1]

Total: 5

2. A curve has intrinsic coordinates (s, ψ) and radius of curvature ρ. [5]

Given that $\rho = 2(s + a)$, where a is constant, show that the intrinsic equation of the curve can be written in the form
$$s = A\mathrm{e}^{2\psi} - a,$$
where A is constant.

3. (a) Prove that [5]
$$\sinh(3x) \equiv 4\sinh^3(x) + 3\sinh(x).$$

(b) Hence, or otherwise, solve the equation [6]
$$\sinh(3x) = 7\sinh^2(x),$$
giving your answers in terms of natural logarithms where appropriate.

Total: 11

4. (a) Find $\displaystyle\int \frac{1}{\sqrt{9 - 4x^2}}\,\mathrm{d}x$. [3]

(b) Find $\displaystyle\int \frac{1 - 2x}{\sqrt{9 - 4x^2}}\,\mathrm{d}x$. [3]

(c) Hence, or otherwise, solve the differential equation [6]
$$\sqrt{9 - 4x^2}\frac{\mathrm{d}y}{\mathrm{d}x} = y(1 - 2x),$$
given that $y = 1$ when $x = 0$.

Total: 12

5. The curve C has equation $y^2 = 4ax$, where a is a positive constant.

(a) Show that an equation of the tangent to C at the point $P(ap^2, 2ap), p \neq 0$, is [4]
$$yp = x + ap^2.$$

The point $Q(aq^2, 2aq)$, is on C where $q \neq 0$ and $p \neq q$. The chord PQ passes through the focus of C. Show that

(b) $pq = -1$, [5]

(c) the tangent to C at P and the tangent to C at Q meet on the directrix of C. [4]

Total: 13

6.
$$I_n = \int_0^{\frac{\pi}{4}} \sec^n(x)\,\mathrm{d}x, \quad n \geq 0.$$

(a) Show that [7]
$$(n-1)I_n = \sqrt{2}^{n-2} + (n-2)I_{n-2}, \quad n \geq 2.$$

(b) Hence find the exact value of I_3, giving your answer in terms of natural logarithms. [6]

Total: 13

7. (a) Show that [9]
$$\int \sqrt{a^2+x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\operatorname{arcsinh}\left(\frac{x}{a}\right) + c.$$

The parametric equations of the curve C are

$$x = 2t, \quad \text{and} \quad y = t^2, \qquad 0 \leq t \leq 3.$$

(b) Show that the length of C is given by [4]
$$2\int_0^2 \sqrt{1+t^2}\,\mathrm{d}t.$$

(c) Find the length of C. [3]

Total: 16