

# Solomon Practice Paper

## Pure Mathematics 5D

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

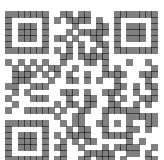
Question	Points	Score
1	5	
2	5	
3	11	
4	12	
5	13	
6	13	
7	16	
Total:	75	

How I can achieve better:

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1.

$$y = \frac{\operatorname{cosech}(x)}{x^2 + 1}.$$

(a) Find  $\frac{dy}{dx}$ . [4]

(b) Find the value of  $\frac{dy}{dx}$  when  $x = 0.5$ , giving your answer to 2 decimal places. [1]

Total: 5

2. A curve has intrinsic coordinates  $(s, \psi)$  and radius of curvature  $\rho$ . [5]

Given that  $\rho = 2(s + a)$ , where  $a$  is constant, show that the intrinsic equation of the curve can be written in the form

$$s = Ae^{2\psi} - a,$$

where  $A$  is constant.

3. (a) Prove that [5]

$$\sinh(3x) \equiv 4 \sinh^3(x) + 3 \sinh(x).$$

(b) Hence, or otherwise, solve the equation [6]

$$\sinh(3x) = 7 \sinh^2(x),$$

giving your answers in terms of natural logarithms where appropriate.

Total: 11

4. (a) Find  $\int \frac{1}{\sqrt{9 - 4x^2}} dx$ . [3]

(b) Find  $\int \frac{1 - 2x}{\sqrt{9 - 4x^2}} dx$ . [3]

(c) Hence, or otherwise, solve the differential equation [6]

$$\sqrt{9 - 4x^2} \frac{dy}{dx} = y(1 - 2x),$$

given that  $y = 1$  when  $x = 0$ .

Total: 12

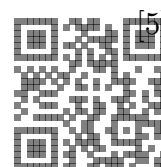
5. The curve  $C$  has equation  $y^2 = 4ax$ , where  $a$  is a positive constant.

(a) Show that an equation of the tangent to  $C$  at the point  $P(ap^2, 2ap)$ ,  $p \neq 0$ , is [4]

$$yp = x + ap^2.$$

The point  $Q(aq^2, 2aq)$ , is on  $C$  where  $q \neq 0$  and  $p \neq q$ . The chord  $PQ$  passes through the focus of  $C$ . Show that

(b)  $pq = -1$ ,



(c) the tangent to  $C$  at  $P$  and the tangent to  $C$  at  $Q$  meet on the directrix of  $C$ . [4]

Total: 13

6.

$$I_n = \int_0^{\frac{\pi}{4}} \sec^n(x) \, dx, \quad n \geq 0.$$

(a) Show that [7]

$$(n - 1)I_n = \sqrt{2}^{n-2} + (n - 2)I_{n-2}, \quad n \geq 2.$$

(b) Hence find the exact value of  $I_3$ , giving your answer in terms of natural logarithms. [6]

Total: 13

7. (a) Show that [9]

$$\int \sqrt{a^2 + x^2} \, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \operatorname{arcsinh} \left( \frac{x}{a} \right) + c.$$

The parametric equations of the curve  $C$  are

$$x = 2t, \quad \text{and} \quad y = t^2, \quad 0 \leq t \leq 3.$$

(b) Show that the length of  $C$  is given by [4]

$$2 \int_0^2 \sqrt{1 + t^2} \, dt.$$

(c) Find the length of  $C$ . [3]

Total: 16

