Solomon Practice Paper

Pure Mathematics 4H

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	9	
4	9	
5	10	
6	15	
7	18	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. (a) Given that $\mathrm{f}(r) = r!$, show that $\mathrm{f}(r+1) - \mathrm{f}(r) = r \times r!$. [2]

   (b) Hence find $\displaystyle\sum_{r=1}^{n} (r \times r!)$. [4]

Total: 6

2. (a) Given that [5]

$$y = \frac{2x}{x^2+9},$$

   express x in terms of y.

   (b) Hence prove that for all real values of x [3]

$$-\frac{1}{a} \leq \frac{2x}{x^2+9} \leq \frac{1}{a},$$

   where a is a positive integer which you should find.

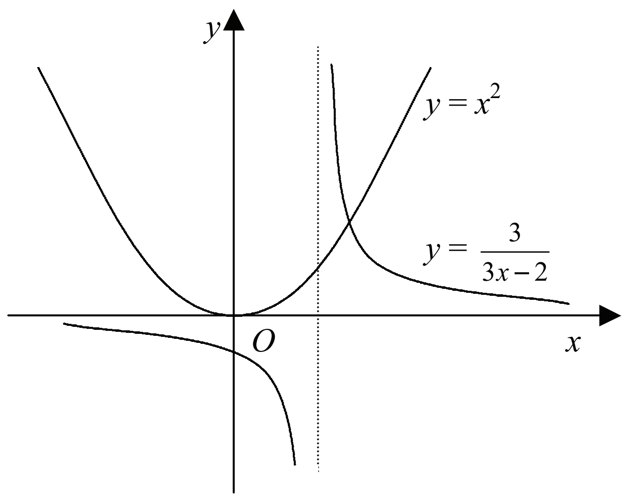
Total: 8

3. Find the general solution of the differential equation [9]

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + xy = 1 - y,$$

   giving your answer in the form $y = \mathrm{f}(x)$.

4. Figure shows part of the curves $y = x^2$ and $y = \dfrac{3}{3x-2}$.



   The curves meet at the point with x-coordinate α.

   (a) Find the integer N such that $\dfrac{N}{10} < \alpha < \dfrac{N+1}{10}$. [4]

   (b) Use interval bisection on the interval found in part (a) to find the value of α correct to 2 decimal places. [5]

Total: 9

5. Given that

$$f(z) \equiv z^4 - 4z^3 + kz^2 - 4z + 13,$$

where k is a real constant, and that $z = \mathbf{i}$ is a solution of the equation $f(z) = 0$,

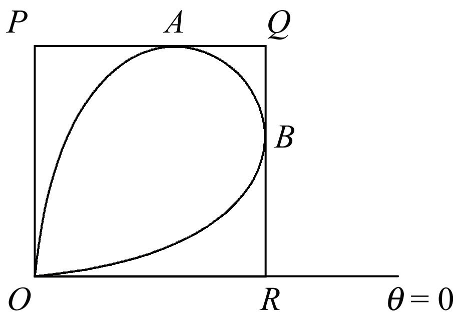
(a) show that $k = 14$, [3]

(b) find all solutions of the equation $f(z) = 0$. [7]

Total: 10

6. The shape of a company logo is to be the region enclosed by the curve with polar equation

$$r^2 = a^2 \sin(2\theta), \quad 0 \le \theta \le \frac{\pi}{2}.$$



A sign in the shape of the logo is to be made by cutting the area enclosed by the curve from a square sheet of metal $OPQR$ where O is the pole and R lies on the initial line, $\theta = 0$, as shown in Figure.

PQ and QR are tangents to the curve, parallel and perpendicular to the initial line respectively, at the points A and B on the curve.

(a) Find the value of θ at the point A. [7]

(b) Show that the area of $OPQR$ is $\frac{3\sqrt{3}}{8}a^2$. [3]

(c) Find the area of the metal sheet which is not used. [5]

Total: 15

7. Given that $x = k\mathrm{e}^{-t}$ satisfies the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 6x = 8\mathrm{e}^{-t},$$

(a) find the value of k. [3]

(b) Hence find the solution of the differential equation for which $x = 1$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 3$ at $t = 0$. [8]

The maximum value of x occurs when $t = T$.

(c) Show that the maximum value of x is $\frac{40}{27}$ and find the value of T. [7]

Total: 18