

# Solomon Practice Paper

## Pure Mathematics 4G

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

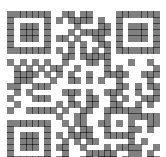
Question	Points	Score
1	7	
2	7	
3	9	
4	11	
5	12	
6	12	
7	17	
Total:	75	

How I can achieve better:

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1. Find the set of values of  $x$  for which

$$\frac{x^2 - 12}{x} \geq 1.$$

[7]

2. Show that the sum of the first  $n$  terms of the series

$$5^2 + 9^2 + 13^2 + 17^2 + \dots$$

[7]

is given by  $\frac{1}{3}n(16n^2 + 36n + 23)$ .

- 3.

$$f(x) \equiv x^3 - 5x^2 + 2.$$

- (a) Show that the equation  $f(x) = 0$  has a root  $\alpha$  in the interval  $[0, 1]$ .

[2]

- (b) Use the Newton-Raphson method with initial value  $x = 0.5$  to find a value for  $\alpha$  which is correct to 2 decimal places.

[5]

- (c) Give a reason why the Newton-Raphson method fails if an initial value of  $x = 0$  is used in part (b).

[2]

Total: 9

4. The complex number  $z$  is given by

$$z = \frac{1 + \mathbf{i}\sqrt{3}}{1 - \mathbf{i}\sqrt{3}}.$$

- (a) Show that  $z$  can be expressed in the form  $\lambda(1 - \mathbf{i}\sqrt{3})$  where  $\lambda$  is a rational number which you should find.

[4]

- (b) Find the modulus and argument of  $z$ .

[3]

- (c) Hence, or otherwise, find the modulus and argument of

[4]

$$\left( \frac{1 + \mathbf{i}\sqrt{3}}{1 - \mathbf{i}\sqrt{3}} \right)^4.$$

Total: 11

5. (a) Find the values of  $p$  and  $q$  such that  $y = p \sin(x) + q \cos(x)$  is a particular integral of the differential equation

[7]

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = \sin(x).$$

- (b) Find the general solution of this differential equation.

[5]

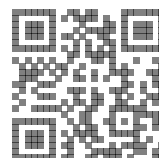
Total: 12

6. (a) Show that

[3]

$$\int 2 \cot(x) dx = \ln(\sin^2(x)) + c,$$

where  $c$  is an arbitrary constant.



- (b) Find the general solution of the differential equation [5]

$$\sin(x) \frac{dy}{dx} + 2y \cos(x) = 1.$$

Given that  $y = 0$  when  $x = \frac{\pi}{4}$ ,

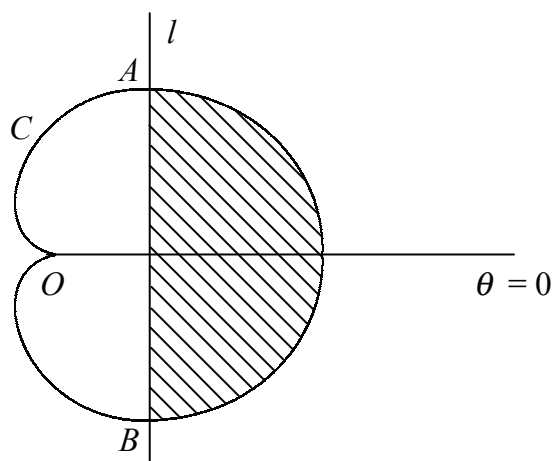
- (c) show that when  $x = \frac{\pi}{3}$ , [4]

$$y = \frac{2}{3} (\sqrt{2} - 1).$$

Total: 12

7. Figure shows the curve  $C$  with polar equation

$$r = 2(1 + \cos(\theta)), \quad -\pi < \theta \leq \pi,$$



and the line  $l$  with polar equation

$$r \cos(\theta) = \frac{3}{2},$$

referred to the pole  $O$  and initial line  $\theta = 0$ .

- (a) Find the polar coordinates of the points  $A$  and  $B$ , where  $l$  intersects  $C$ . [6]

- (b) Show that the area of triangle  $OAB$  is  $\frac{9\sqrt{3}}{4}$ . [3]

- (c) Hence find the area of the shaded region bounded by  $C$  and  $l$ . [8]

Total: 17

