Solomon Practice Paper

Pure Mathematics 4E

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	7	
2	7	
3	9	
4	10	
5	12	
6	13	
7	17	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. The complex number w is given by $w = \dfrac{10+5\mathbf{i}}{2-\mathbf{i}}$.

   (a) Express w in the form $a + \mathbf{i}b$ where a and b are real. [3]

   (b) Using your answer to part (a) find the complex number z such that [4]

   $$z + 2z^{\star} = w.$$

Total: 7

2. Show that [7]

$$\sum_{r=0}^{n}(r+1)(r+2) = \frac{1}{3}(n+1)(n+2)(n+3).$$

3. Find the equation of the curve which passes through the origin and for which [9]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x + y,$$

giving your answer in the form $y = \mathrm{f}(x)$.

4. The curve C has the polar equation

$$r = a(1+\sin(\theta)), \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

   (a) Sketch the curve C. [2]

   (b) Find the polar coordinates of the point on the curve where the tangent to the curve is perpendicular to the initial line $\theta = 0$. [8]

Total: 10

5. (a) Find, in terms of a and b, the equations of the asymptotes to the curve with equation [3]

   $$y = \frac{ax-1}{x+b},$$

   where a and b are positive constants.

   (b) Sketch the curve [3]

   $$y = \frac{ax-1}{x+b},$$

   showing the coordinates of any points of intersection with the coordinate axes.

   (c) Hence, or otherwise, find the set of values of x for which [6]

   $$\left|\frac{3x-1}{x+2}\right| < 2.$$

Total: 12

6. (a) Show that the equation $e^x - 4\sin(x) = 0$ has a root, α, in the interval $[0, 1]$ and a root, β, in the interval $[1, 1.5]$. [3]

   (b) Using the Newton-Raphson method with an initial value of $x = 0.5$, find α correct to 2 decimal places. [5]

   (c) Use linear interpolation once between the values $x = 1$ and $x = 1.5$ to find an approximate value for β, giving your answer correct to 1 decimal place. [3]

   (d) Determine whether or not your answer to part (c) gives the value of β correct to 1 decimal place. [2]

Total: 13

7. (a) Given that y is a function of t and that $x = t^{\frac{1}{2}}$, where $x > 0$, show that [6]

   i. $\frac{dy}{dx} = 2t^{\frac{1}{2}}\frac{dy}{dt}$,

   ii. $\frac{d^2y}{dx^2} = 2\frac{dy}{dt} + 4t\frac{d^2y}{dt^2}$.

   (b) Use your answers to part (a) to show that the substitution $x = t^{\frac{1}{2}}$ transforms the differential equation [4]

$$\frac{1}{x^2}\frac{d^2y}{dx^2} + \left(\frac{4}{x} - \frac{1}{x^3}\right)\frac{dy}{dx} + 3y = 3x^2 + 5 \qquad (\star)$$

   into the differential equation

$$4\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 3y = 3t + 5.$$

   (c) Hence find the general solution of differential equation $\star$. [7]

Total: 17