Solomon Practice Paper

Pure Mathematics 4D

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	7	
2	8	
3	8	
4	10	
5	10	
6	15	
7	17	
Total:	75	

How I can achieve better:

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1. The function f is defined by

$$\mathrm{f}(x) \equiv 3x^3 + kx^2 + 42x + k,$$

where k is an integer.

Given that $(3 + \mathbf{i})$ is a root of the equation $\mathrm{f}(x) = 0$,

(a) find a quadratic factor of $\mathrm{f}(x)$, [3]

(b) find the value of k. [4]

Total: 7

2. Find the set of values of x for which [8]

$$\frac{x}{x-1} > \frac{2}{3-x}.$$

3. Given that $y = \dfrac{1}{2}$ when $x = 0$, solve the differential equation [8]

$$\frac{\mathrm{d}y}{\mathrm{d}x} - 3x + 4xy = 0,$$

giving your answer in the form $y = \mathrm{f}(x)$.

4. (a) Express $\dfrac{3r+4}{r(r+1)(r+2)}$ in partial fractions. [3]

(b) Hence, show that [7]

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)} = \frac{n(5n+9)}{2(n+1)(n+2)}.$$

Total: 10

5. (a) Find the values of a, b and c such that $y = ax^2 + bx + c$ satisfies the differential equation [5]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 2\frac{\mathrm{d}y}{\mathrm{d}x} + 10y = 5x^2 - 13x + 1.$$

(b) Hence, find the general solution of this differential equation. [5]

Total: 10

6.

$$\mathrm{f}(x) \equiv \frac{2}{3}x + \sin(2x) - 1, \quad x \in \mathbb{R}.$$

(a) By sketching the graphs of $y = \sin(2x)$ and $y = 1 - \frac{2}{3}x$ on the same diagram, find the number of solutions to the equation $\mathrm{f}(x) = 0$. [3]

(b) i. Show that one root, α, of the equation $\mathrm{f}(x) = 0$ lies in the interval $(2.5, 3)$. [7]

ii. Use one application of the method of linear interpolation on this interval to find an approximate value for α, giving your answer correct to 2 decimal places.

iii. Determine whether or not your answer to part (ii) gives the value of α correct to 2 decimal places.

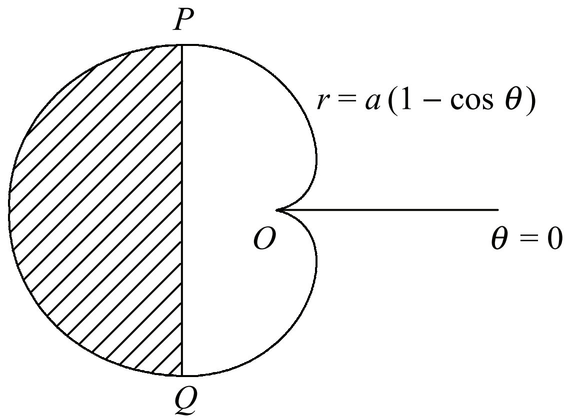
(c) Use the Newton-Raphson method with a starting value of $x = 0.5$ to find another root of the equation $f(x) = 0$ correct to 3 significant figures. [5]

Total: 15

7. Figure shows the curve C with polar equation

$$r = a(1 - \cos(\theta)), \quad 0 \leq \theta < 2\pi,$$

where a is a positive constant.



At the points P and Q the tangents to the curve are parallel to the initial line $\theta = 0$.

(a) Find the polar coordinates of P and Q. [7]

The shaded region is bounded by the curve C and the straight line PQ.

(b) Show that the area of the shaded region is $\frac{1}{16}a^2\left(8\pi + 9\sqrt{3}\right)$. [10]

Total: 17