Solomon Practice Paper

Pure Mathematics 4B

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	9	
4	9	
5	10	
6	15	
7	18	
Total:	75	

How I can achieve better:

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1. Find the set of values of x for which [6]

$$|2x^2 - 5x| < x.$$

2. (a) Sketch the curve C with the polar equation [3]

$$r^2 = a^2 \sin^2(2\theta), \quad 0 \leq \theta < 2\pi.$$

(b) Find the exact area of the region enclosed by one loop of the curve C. [5]

Total: 8

3. (a) Show that [6]

$$\sum_{r=1}^{n} (r^2+1)(r-1) = \frac{1}{12} n(n-1)(3n^2+5n+8).$$

(b) Hence evaluate [3]

$$\sum_{r=5}^{25} (r^2+1)(r-1).$$

Total: 9

4. (a) Find the general solution of the differential equation [6]

$$\frac{\mathrm{d}y}{\mathrm{d}x} - y\cot(x) = \sin(2x).$$

(b) Given also that $y = 2$ when $x = \dfrac{\pi}{6}$, find the exact value of y when $x = \frac{2\pi}{3}$. [3]

Total: 9

5.

$$\mathrm{f}(x) \equiv x^3 - \ln\left(4 - x^2\right), \quad x \in \mathbb{R}, \quad -2 < x < 2.$$

(a) Show that one root, α, of the equation $\mathrm{f}(x) = 0$ lies in the interval $1.0 < \alpha < 1.1$. [2]

(b) Starting with $x = 1.0$, show that using the Newton-Raphson method twice gives an approximation to α that is correct to 6 decimal places. [8]

Total: 10

6. The complex numbers z_1, z_2 and z_3 are given by

$$z_1 = 7 - \mathbf{i}, \quad z_2 = 1 + \mathbf{i}\sqrt{3}, \quad z_3 = a + \mathbf{i}b,$$

where a and b are rational constants.

Given that the modulus of $z_1 z_3$ is 50,

(a) find the modulus of z_3. [3]

Given also that the argument of $\frac{z_2}{z_3}$ is $\frac{7\pi}{12}$,

(b) find the argument of z_3. [3]

(c) Find the values of a and b. [2]

(d) Show that $\frac{z_1}{z_3} = \frac{1}{5}(4+3\mathbf{i})$. [3]

(e) Represent z_1, z_3 and $\frac{z_1}{z_3}$ on the same Argand diagram. [2]

(f) By considering the modulus and argument of z_1 and z_3, explain why [2]

$$\frac{z_3}{z_1} = \left(\frac{z_1}{z_3}\right)^{\star}.$$

Total: 15

7. (a) Given that $x = \mathrm{e}^t$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\frac{\mathrm{d}y}{\mathrm{d}t}$ and show that [5]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \mathrm{e}^{-2t}\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - \frac{\mathrm{d}y}{\mathrm{d}t}\right).$$

(b) Show that the substitution $x = \mathrm{e}^t$ transforms the differential equation [3]

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 6x^2$$

into the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 2\frac{\mathrm{d}y}{\mathrm{d}t} - 3y = 6\mathrm{e}^{2t}.$$

(c) Given that when $x = 1, y = 3$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -5$, solve the differential equation [10]

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - x\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 6x^2.$$

Total: 18