

Solomon Practice Paper

Pure Mathematics 4B

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

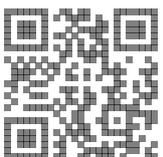
| Question | Points | Score |
|----------|--------|-------|
| 1 | 6 | |
| 2 | 8 | |
| 3 | 9 | |
| 4 | 9 | |
| 5 | 10 | |
| 6 | 15 | |
| 7 | 18 | |
| Total: | 75 | |

How I can achieve better:

-
-
-



Last updated: May 5, 2023



1. Find the set of values of x for which [6]

$$|2x^2 - 5x| < x.$$

2. (a) Sketch the curve C with the polar equation [3]

$$r^2 = a^2 \sin^2(2\theta), \quad 0 \leq \theta < 2\pi.$$

- (b) Find the exact area of the region enclosed by one loop of the curve C . [5]

Total: 8

3. (a) Show that [6]

$$\sum_{r=1}^n (r^2 + 1)(r - 1) = \frac{1}{12}n(n - 1)(3n^2 + 5n + 8).$$

- (b) Hence evaluate [3]

$$\sum_{r=5}^{25} (r^2 + 1)(r - 1).$$

Total: 9

4. (a) Find the general solution of the differential equation [6]

$$\frac{dy}{dx} - y \cot(x) = \sin(2x).$$

- (b) Given also that $y = 2$ when $x = \frac{\pi}{6}$, find the exact value of y when $x = \frac{2\pi}{3}$. [3]

Total: 9

5.

$$f(x) \equiv x^3 - \ln(4 - x^2), \quad x \in \mathbb{R}, \quad -2 < x < 2.$$

- (a) Show that one root, α , of the equation $f(x) = 0$ lies in the interval $1.0 < \alpha < 1.1$. [2]

- (b) Starting with $x = 1.0$, show that using the Newton-Raphson method twice gives an approximation to α that is correct to 6 decimal places. [8]

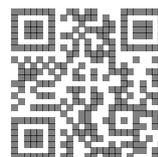
Total: 10

6. The complex numbers z_1, z_2 and z_3 are given by

$$z_1 = 7 - \mathbf{i}, \quad z_2 = 1 + \mathbf{i}\sqrt{3}, \quad z_3 = a + \mathbf{i}b,$$

where a and b are rational constants.

Given that the modulus of $z_1 z_3$ is 50,



(a) find the modulus of z_3 . [3]

Given also that the argument of $\frac{z_2}{z_3}$ is $\frac{7\pi}{12}$,

(b) find the argument of z_3 . [3]

(c) Find the values of a and b . [2]

(d) Show that $\frac{z_1}{z_3} = \frac{1}{5}(4 + 3i)$. [3]

(e) Represent z_1, z_3 and $\frac{z_1}{z_3}$ on the same Argand diagram. [2]

(f) By considering the modulus and argument of z_1 and z_3 , explain why [2]

$$\frac{z_3}{z_1} = \left(\frac{z_1}{z_3}\right)^*$$

Total: 15

7. (a) Given that $x = e^t$, find $\frac{dy}{dx}$ in terms of $\frac{dy}{dt}$ and show that [5]

$$\frac{d^2y}{dx^2} = e^{-2t} \left(\frac{d^2y}{dt^2} - \frac{dy}{dt} \right).$$

(b) Show that the substitution $x = e^t$ transforms the differential equation [3]

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = 6x^2$$

into the differential equation

$$\frac{d^2y}{dt^2} - 2 \frac{dy}{dt} - 3y = 6e^{2t}.$$

(c) Given that when $x = 1, y = 3$ and $\frac{dy}{dx} = -5$, solve the differential equation [10]

$$x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = 6x^2.$$

Total: 18

