**Solomon Practice Paper**

**Pure Mathematics 3I**

**Time allowed:** 90 **minutes**

**Centre:** www.CasperYC.club

**Name:**

**Teacher:**

| Question | Points | Score |
|----------|--------|-------|
| 1 | 5 | |
| 2 | 7 | |
| 3 | 9 | |
| 4 | 10 | |
| 5 | 11 | |
| 6 | 16 | |
| 7 | 17 | |
| Total: | 75 | |

**How I can achieve better:**

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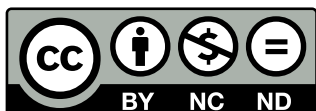
Last updated: May 5, 2023
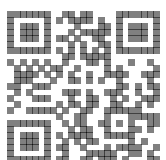
1. [5]

$$f(x) \equiv x^2 + 5x + 26.$$

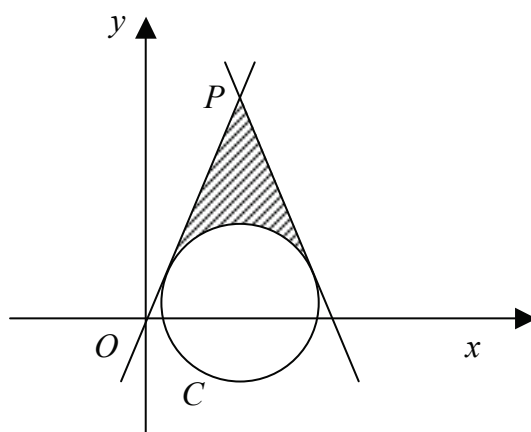The remainder when $f(x)$ is divided by $(x - a)$ is twice the remainder when $f(x)$ is divided by $(x + a)$.

Find the possible values of $a$.

2. Given that $y = \dfrac{\pi}{8}$ when $x = \dfrac{\pi}{4}$. Solve the differential equation [7]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(2x)\cot^2(2y).$$

3. Figure shows the circle $C$ with equation

$$x^2 + y^2 - 12x - 2y + 12 = 0.$$



(a) Find the coordinates of the centre of $C$ and write down its radius. [4]

The shaded region in the diagram is enclosed by $C$ and the two tangents to $C$ which pass through the point $P$ with coordinates $(6, 14)$.

(b) Show that the area of the shaded region is 30.6, correct to 3 significant figures. [5]

Total: 9

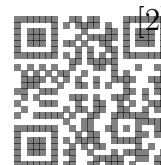4. Using the substitution $u = \mathrm{e}^x - 1$, show that [10]

$$\int_{\ln(2)}^{\ln(5)} \frac{\mathrm{e}^{2x}}{\sqrt{\mathrm{e}^x - 1}} \, \mathrm{d}x = \frac{20}{3}.$$

5. Relative to a fixed origin, $O$, the points $A$, B and $C$ have position vectors $(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$, $(10\mathbf{i} - 5\mathbf{j} - 2\mathbf{k})$ and $(4\mathbf{i} + \mathbf{j} - 5\mathbf{k})$ respectively.

(a) Show that $\overrightarrow{AC} = k\overrightarrow{AB}$, where $k$ is an exact fraction. [3]

(b) Find $\cos(\angle OAC)$. [5]

(c) Find the area of triangle $OAC$ correct to 4 significant figures. [2]

(d) Hence, write down the area of triangle $OAB$ correct to 3 significant figures. [1]

Total: 11

6. (a) Find the values of $A$ and $B$ for which [3]

$$\frac{x-7}{(x+2)(x-1)} \equiv \frac{A}{x+2} + \frac{B}{x-1}.$$

(b) Show that [5]

$$\int_2^4 \frac{x-7}{(x+2)(x-1)} \, \mathrm{d}x = \ln\left(\frac{3}{8}\right).$$

(c) Find, for $|x| < 1$, the series expansion of [8]
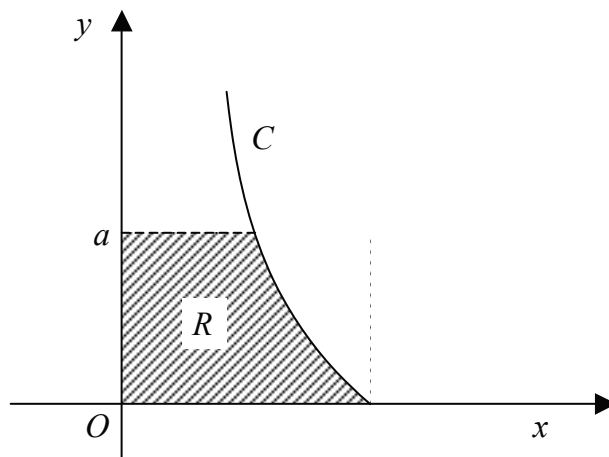
$$\frac{x-7}{(x+2)(x-1)}$$

up to and including the term in $x^2$, giving the coefficients as exact fractions.

Total: 16

7. Figure shows the curve $C$ which is defined by the parametric equations

$$x = 2a\cos(t), \quad \text{and} \quad y = a\tan^2(t), \quad 0 \le t \le \frac{\pi}{2},$$
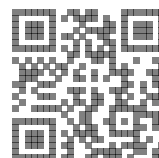
where $t$ is a parameter and $a$ is a positive constant.

(a) Find and simplify an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $t$. [5]

The point $P$ on $C$ has parameter $t = \dfrac{\pi}{3}$

(b) Find an equation of the tangent to $C$ at the point $P$. [4]

(c) Show that a Cartesian equation of $C$ is [3]

$$x^2 = \frac{4a^3}{a+y}.$$

The shaded region $R$ is bounded by $C$, the positive coordinate axes, and the line $y = a$.

(d) Find the volume of the solid generated when the region $R$ is rotated through $2\pi$ radians [5]
about the $y$-axis.

Total: 17