Solomon Practice Paper

Pure Mathematics 3H

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	6	
3	7	
4	9	
5	10	
6	10	
7	12	
8	15	
Total:	75	

Centre: **www.CasperYC.club**

Name:

Teacher:

How I can achieve better:

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1. In the series expansion of $(1+2x)^k$, for $|x| < \frac{1}{2}$, the coefficient of x^2 is 24.

   (a) Find the two possible values of k. [4]

   Given that $k < 0$,

   (b) find the coefficient of x^3 in the expansion. [2]

   Total: 6

2. Use integration by parts to evaluate [6]

$$\int_0^{\frac{\pi}{2}} x\cos(x)\,\mathrm{d}x,$$

   giving your answer in terms of π.

3.

$$\mathrm{f}(x) \equiv \frac{x-11}{(x+4)(x-2)}.$$

   (a) Express $\mathrm{f}(x)$ in the form [3]

$$\frac{A}{x+4} + \frac{B}{x-2}.$$

   (b) Evaluate $\mathrm{f}'(1)$, giving your answer as an exact fraction. [4]

   Total: 7

4. The functions f and g are defined by [9]

$$\begin{aligned} \mathrm{f}\colon x &\mapsto (x-2)^2, \\ \mathrm{g}\colon x &\mapsto ax+b, \end{aligned}$$

   where a and b are integer constants.

   Given that when $\mathrm{fg}(x)$ is divided by $(x-1)$ the remainder is 1 and that $(2x-3)$ is a factor of $\mathrm{gf}(x)$, find the values of a and b.

5. Relative to a fixed origin, O, the points A and B have position vectors $(\mathbf{i}+2\mathbf{j}-6\mathbf{k})$ and $(15\mathbf{i}+9\mathbf{j}+\mathbf{k})$ respectively.

   (a) Find, in vector form, an equation of the line AB. [3]

   The point C has position vector $(5\mathbf{i}+\mathbf{j}+2\mathbf{k})$.

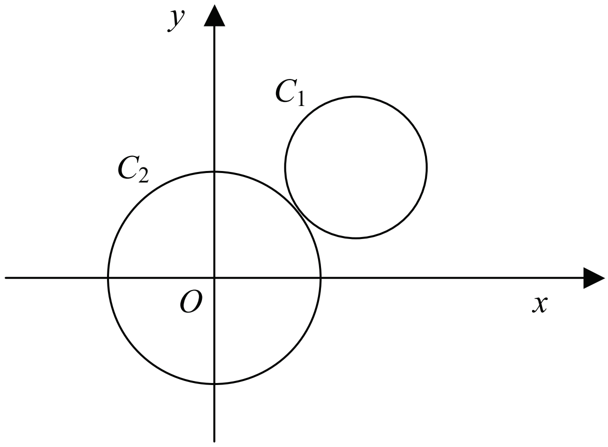
   (b) Find the length AC. [1]

   The point D lies on the line AB such that $\angle ADC = \angle DAC$.

   (c) Find the position vector of the point D. [6]

   Total: 10

6. Figure shows the circles C_1 and C_2.



Circle C_1 has the equation

$$x^2 + y^2 - 16x - ky + 84 = 0,$$

where k is a positive constant.

(a) Find in terms of k [5]

i. the coordinates of the centre of C_1,

ii. the radius of C_1.

Circle C_2 has the equation

$$x^2 + y^2 - 36 = 0.$$

Given that circles C_1 and C_2 are touching,

(b) find the value of k. [5]

Total: 10

7. A computer screen saver program generates a coloured region of random size and shape. This region then expands until it fills the screen. A new region of a different colour is then formed.

The program is written so that the rate at which the area of the region increases is proportional to its current area.

(a) By forming and solving a differential equation, show that t seconds after it is formed the area, A cm^2, of the region is given by $A = A_0 e^{kt}$, where A_0 is the initial area of the region in cm^2 and k is a constant. [6]

Given that once formed the area of a region increases by 50% in 0.4 seconds,

(b) find the value of k correct to 4 significant figures. [3]

A coloured region of area 3.6 cm^2 is generated on a screen measuring 24 cm by 32 cm.

(c) Find, in seconds correct to 1 decimal place, how long it takes for the region to fill the screen. [3]

Total: 12

8. A curve is defined parametrically by

$$x = \frac{2t}{1+t}, \quad \text{and} \quad y = \frac{t^2}{1+t}, \quad t \neq -1.$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [5]

The point P on the curve has coordinates $(1, \frac{1}{2})$.

(b) Show that the normal to the curve at P has the equation [5]

$$4x + 6y - 7 = 0.$$

The normal to the curve at P meets the curve again at the point Q.

(c) Find the coordinates of Q. [5]

Total: 15