**Solomon Practice Paper**

**Pure Mathematics 3E**

**Time allowed:** 90 **minutes**

**Centre:** **www.CasperYC.club**

**Name:**

**Teacher:**

| Question | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 6 | |
| 3 | 7 | |
| 4 | 9 | |
| 5 | 10 | |
| 6 | 11 | |
| 7 | 12 | |
| 8 | 15 | |
| Total: | 75 | |

**How I can achieve better:**

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Last updated: May 5, 2023

1. (a) Expand [4]

$$\left(1+\frac{2}{3}x\right)^{-2}$$

in ascending powers of $x$ as far as the term in $x^3$.

(b) State the set of values of $x$ for which your expansion is valid. [1]

Total: 5

2. Given that $y = -1$ when $x = 1$, solve the differential equation [6]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y^2}{x^3},$$

giving your answer in the form $y = \mathrm{f}(x)$.

3.

$$\mathrm{f}(x) \equiv x^3 + ax^2 + bx - 3.$$

Given that when $\mathrm{f}(x)$ is divided by $(x+1)$ the remainder is 2,

(a) find a linear relationship between $a$ and $b$. [2]

Given also that $(3x-2)$ is a factor of $\mathrm{f}'(x)$,

(b) find the values of $a$ and $b$. [5]

Total: 7

4.

$$\mathrm{f}(x) \equiv \frac{3x^2-4x-1}{(x-2)(x+1)}.$$

(a) Express $\mathrm{f}(x)$ in the form [4]

$$A + \frac{B}{x-2} + \frac{C}{x+1}.$$

(b) Show that [5]

$$\int_3^5 \mathrm{f}(x)\,\mathrm{d}x = 6 + \ln\left(\frac{4}{3}\right).$$

Total: 9

5. A circle has the equation

$$x^2 + y^2 + 3x - 6y + 5 = 0.$$

(a) Find the distance of the centre of the circle from the origin in the form $k\sqrt{5}$ where $k$ is an exact fraction. [5]

(b) Show that the line with equation [5]

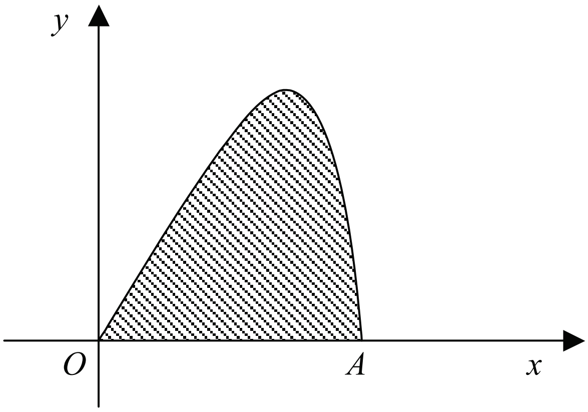
$$3x - 4y + 4 = 0$$

is a tangent to the circle.

Total: 10

6. Figure shows the curve with parametric equations

$$x = \cos(t), \quad \text{and} \quad y = 3\sin(2t), \quad 0 \le t \le \frac{\pi}{2}.$$




The curve meets the $x$-axis at the origin, $O$, and at the point $A$ with coordinates $(1, 0)$.

(a) Find the value of the parameter $t$ at the points $O$ and $A$. [3]

(b) Find the area of the shaded region enclosed by the curve and the $x$-axis. [8]

Total: 11

7. A curve is given by the equation

$$y = 4e^{2x} + e^{-x}.$$

(a) Find in exact form the coordinates of the stationary point on the curve. [9]

(b) Sketch the curve, labelling the coordinates of any points of intersection with the coordinate axes. [3]

Total: 12

8. The lines $l_1$ and $l_2$ are given by

$$\begin{aligned} l_1 &: \mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 9\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k}), \\ l_2 &: \mathbf{r} = 9\mathbf{i} + 2\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} + 7\mathbf{k}). \end{aligned}$$

(a) Show that the lines $l_1$ and $l_2$ intersect and find the position vector of their point of intersection, $P$. [5]

(b) Show that the acute angle, $\theta$, between lines $l_1$ and $l_2$ satisfies [4]

$$\cos(\theta) = \frac{1}{3}\sqrt{6}.$$

The point $Q$ lies in the plane containing lines $l_1$ and $l_2$ and has position vector $(4\mathbf{i} + \mathbf{j} + 3\mathbf{k})$.

(c) Find $\cos(\alpha)$, where $\alpha$ is the acute angle between $PQ$ and line $l_1$. [3]

(d) By finding $\cos(2\theta)$, prove that $\alpha = 2\theta$. [3]

Total: 15