Solomon Practice Paper

Pure Mathematics 3C

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	4	
2	7	
3	8	
4	9	
5	10	
6	11	
7	13	
8	13	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023
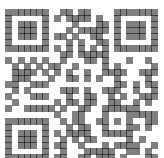
1. [4]

$$\mathrm{f}(x) \equiv 2x^3 + kx^2 - 2k^2x + 9.$$

When f(x) is divided by $(x - 2)$ the remainder is 1. Find the two possible values of k.

2. Given that $y = (2x + 3)\mathrm{e}^{-2x}$,

   (a) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, [3]

   (b) show that [4]
   $$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 0.$$

   Total: 7

3. The binomial expansion of $(1 + ax)^b$ in ascending powers of x as far as the term in x^2 is

   $$1 - 10x + 75x^2 + \dots, \quad |ax| < 1.$$

   (a) Find the values of the constants a and b. [6]

   (b) Find the coefficient of x^3 in the expansion. [2]

   Total: 8

4. Relative to a fixed origin, O, the points P and Q have position vectors $(2\mathbf{i} + 3\mathbf{j} + 8\mathbf{k})$ and $(6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k})$ respectively.

   (a) Find, in vector form, an equation of the line l which passes through the points P and Q. [3]

   The line m has the equation

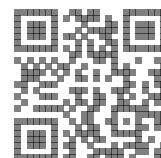 $$\mathbf{r} = 8\mathbf{i} + 6\mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}).$$

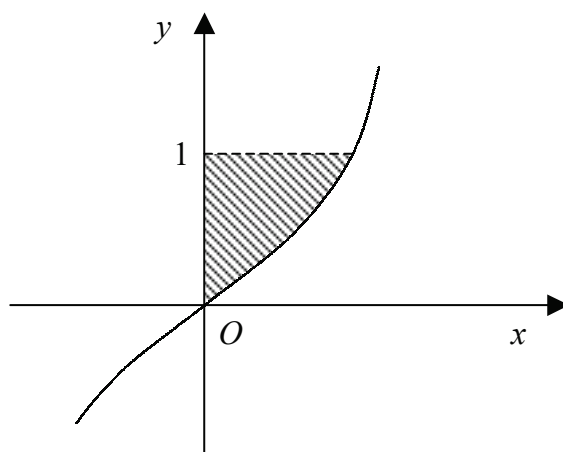   (b) Show that the point P lies on the line m. [3]

   (c) Show that the lines l and m are perpendicular. [3]

   Total: 9

5. Figure shows the curve with parametric equations

   $$x = 2\sin(t), \quad \text{and} \quad y = \tan(t), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$
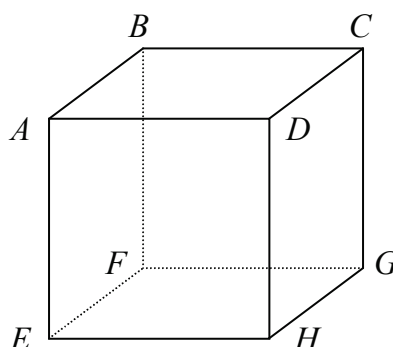
(a) Find the value of t at the point with coordinates $(\sqrt{2}, 1)$.   [2]

The shaded region is enclosed by the curve, the line $y = 1$ and the y-axis.

(b) Use integration to show that the area of the shaded region is $2(\sqrt{2} - 1)$.   [8]

Total: 10

6. Figure below shows a cube.

The points A and G are diagonally opposite corners of the cube and have position vectors $(\mathbf{i} + 8\mathbf{j} - 3\mathbf{k})$ and $(8\mathbf{i} - 9\mathbf{j} + 10\mathbf{k})$ respectively relative to a fixed origin, O.

(a) Show that the length of one edge of the cube is 13.   [4]

(b) By finding the distance of the centre of the cube from O, prove that O is inside the cube.   [7]
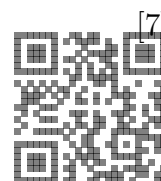
Total: 11

7.
$$\mathrm{f}(x) \equiv \frac{3}{(x-1)(x+2)}, \quad x \in \mathbb{R}, \quad x \neq -2, 1.$$

(a) Find the values of A and B for which   [3]

$$\mathrm{f}(x) \equiv \frac{A}{x-1} + \frac{B}{x+2}.$$

(b) Find the coordinates of the stationary point on the curve $y = \mathrm{f}(x)$.   [7]

   (c) Sketch the curve $y = \mathrm{f}(x)$. [3]

Total: 13

8. A hot oven is turned off and allowed to cool with the door shut.

   Let θ be the excess temperature inside the oven over the temperature of the air outside the oven. The rate at which θ decreases is proportional to θ.

   (a) By forming and solving a differential equation, show that [5]

$$\theta = A\mathrm{e}^{-kt},$$

     where t is the time in minutes after the oven is switched off and A and k are constants.

   The temperature inside the oven is $220°$ C when it is turned off. After 20 minutes the temperature inside the oven is $140°$ C.

   Assuming that the temperature outside the oven remains constant at $20°$ C as the oven cools,

   (b) find the value of A and the value of k correct to 3 significant figures, [5]

   (c) find to the nearest minute, how much longer it takes for the temperature inside the oven to fall to $60°$ C. [3]

Total: 13