Solomon Practice Paper

Pure Mathematics 3A

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	7	
3	8	
4	8	
5	9	
6	11	
7	13	
8	13	
Total:	75	

How I can achieve better:

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1. A curve has the equation $y = x^2 e^{3x}$.

   (a) Find and simplify an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

   (b) Find the coordinates of any stationary points on the curve. [3]

   Total: 6

2. $$\mathrm{f}(x) \equiv x^3 + ax + 2.$$

   Given that the remainder when $\mathrm{f}(x)$ is divided by $(x+2)$ is the same as the remainder when $\mathrm{f}(x)$ is divided by $(x-3)$,

   (a) find the value of a, [4]

   (b) find as an exact fraction the remainder when $\mathrm{f}(x)$ is divided by $(2x-5)$. [3]

   Total: 7

3. (a) Expand $(1+2x)^{\frac{1}{2}}$ in ascending powers of x as far as the term in x^3, simplifying each coefficient. [4]

   (b) State the set of values of x for which your series is valid. [1]

   (c) Use your series with a suitable value of x to estimate the value of $\sqrt{1.02}$ correct to 6 significant figures. [3]

   Total: 8

4. $$\mathrm{f}(x) \equiv \frac{5}{(3x-2)(x+1)}.$$

   (a) Express $\mathrm{f}(x)$ in the form [3]
   $$\frac{A}{3x-2} + \frac{B}{x+1}.$$

   (b) Show that [5]
   $$\int_2^4 \mathrm{f}(x)\,\mathrm{d}x = \ln\left(\frac{3}{2}\right).$$

   Total: 8

5. The circle C has the equation
   $$x^2 + y^2 - 4x - 10y + 20 = 0.$$

   (a) Find the coordinates of the centre of C and write down its radius. [5]

   (b) Find an equation for the smallest circle that touches both the circle C and the x-axis. [4]

   Total: 9

6. A curve has the equation
   $$2x^2y - 6y + x^3 = 2.$$

(a) Show that [5]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3x^2 + 4xy}{6 - 2x^2}.$$

The point A with coordinates $(2, k)$ lies on the curve.

(b) Find the value of k. [2]

(c) Show that the normal to the curve at A has the equation [4]

$$x + 6y + 16 = 0.$$

Total: 11

7. (a) Using the substitution $u = \sin(x)$, or otherwise, find [4]

$$\int \cos(x)\sin^2(x)\,\mathrm{d}x.$$

(b) Hence, find [3]

$$\int \cos^3(x)\,\mathrm{d}x.$$

(c) Given that $y = \frac{\pi}{4}$ when $x = \frac{\pi}{6}$, solve the differential equation [6]

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos^2(y)\cos^3(x).$$

Total: 13

8. Relative to a fixed origin, O, the points A and B have position vectors $(7\mathbf{i} - 7\mathbf{j} + 5\mathbf{k})$ and $(\mathbf{i} - 6\mathbf{j} + 12\mathbf{k})$ respectively.

(a) Find, in vector form, an equation of the line l which passes through A and B. [3]

Given that the point C has position vector $(-3\mathbf{i} + 12\mathbf{j} + 10\mathbf{k})$ and that M is the mid-point of BC,

(b) find the position vector of the point M. [3]

Given also that $ABMD$ is a rhombus,

(c) show that the position vector of the point D is $(5\mathbf{i} + 2\mathbf{j} + 4\mathbf{k})$, [2]

(d) find the area of $ABMD$ in the form $k\sqrt{2}$ where k is an integer. [5]

Total: 13