Solomon Practice Paper

Pure Mathematics 2I

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	5	
2	5	
3	9	
4	9	
5	11	
6	11	
7	12	
8	13	
Total:	75	

How I can achieve better:

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1. Express $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ [5]

$$\frac{2}{x-2} + \frac{3x}{x^2-4} - \frac{5}{x+2}$$

   as a single fraction in its simplest form.

2. (a) Find [2]
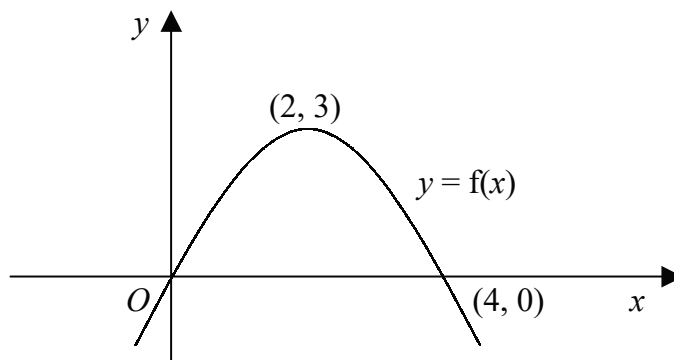
$$\int e^x + 2x + 1 \, dx.$$

   (b) Evaluate [3]

$$\int_0^2 e^x + 2x + 1 \, dx,$$

   giving your answer in terms of e.

Total: 5

3. Figure shows part of the curve $y = f(x)$ which meets the x-axis at the origin, O, and at the point with coordinates $(4, 0)$. The curve has a maximum point with coordinates $(2, 3)$.

   Showing the coordinates of any turning points and any points where each curve meets the x-axis, sketch on separate diagrams graphs of

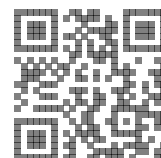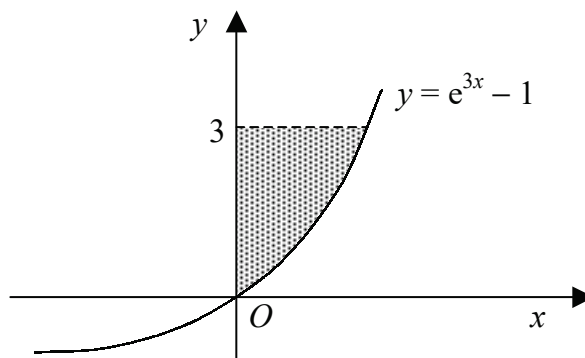   (a) $y = |f(x)|$, [2]

   (b) $y = f\left(\frac{1}{2}x\right)$, [3]

   (c) $y = f(|x|)$. [4]

Total: 9

4. Figure shows part of the curve $y = e^{3x} - 1$.

(a) Write the equation of the curve in the form $x = \mathrm{f}(y)$. [3]

The shaded region is enclosed by the curve, the y-axis and the line $y = 3$.

(b) Show that using the trapezium rule with 3 intervals of equal width gives an estimate of $\frac{1}{3}\left(2\ln(2) + \ln(3)\right)$ for the area of the shaded region. [6]

Total: 9

5. A sequence is defined by the following recurrence relation:

$$u_{n+1} = \frac{2}{u_n} - k, \quad n \geq 1, \quad u_1 = \frac{1}{2}.$$

(a) Find expressions in terms of k for u_2 and u_3. [3]

Given that $u_3 = 7u_2$,

(b) find the two possible values of k. [5]

Given also that k is an integer,

(c) show that $u_4 = -\dfrac{37}{7}$. [3]

Total: 11

6. (a) Find the values of R and α, where x is measured in degrees, $R > 0$, and $0 < \alpha < 90°$, for which [5]

$$\cos(x) - \sqrt{3}\sin(x) \equiv R\cos(x + \alpha).$$

(b) Hence, find the values of x in the interval $0 \leq x \leq 360°$, for which [6]

$$\cos(x) - \sqrt{3}\sin(x) \equiv 2\cos(x + 30°).$$

Total: 11

7. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}\colon x &\mapsto x^2 - 4, \quad x \in \mathbb{R}, \\ \mathrm{g}\colon x &\mapsto 2x + 1, \quad x \in \mathbb{R}. \end{aligned}$$
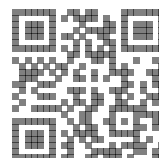
(a) State the range of f. [1]

(b) Define fg as simply as possible. [3]

(c) Solve the equation $\mathrm{fg}(x) = 0$. [2]

(d) Prove that there are no real values of x for which $\mathrm{fg}(x) = \mathrm{gf}(x)$. [6]

Total: 12

8. Figure shows the curve with equation $y = \mathrm{e}^x - 3x$
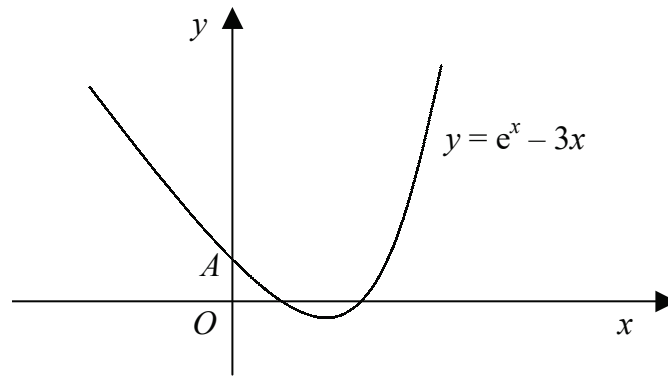
which meets the y-axis at the point A.

(a) Find an equation of the normal to the curve at A. [7]

The point B lies on the curve and has coordinates $(\ln(5), 5 - 3\ln(5))$.

(b) Find an equation of the normal to the curve at B. [4]

The normals to the curve at A and B intersect at the point C.

(c) Show that the x-coordinate of C is $\left(4 - \dfrac{5}{2}\ln(5)\right)$. [2]

Total: 13