Solomon Practice Paper

Pure Mathematics 2G

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	5	
2	7	
3	8	
4	8	
5	8	
6	10	
7	13	
8	16	
Total:	75	

How I can achieve better:

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Last updated: May 5, 2023

1. The terms of a sequence satisfy the following recurrence relation:

$$u_{n+1} = \frac{u_n - 1}{2}, \quad n \geq 1.$$

Given that $u_4 = \frac{1}{4}$, find the value of

(a) u_5, [2]

(b) u_1. [3]

Total: 5

2. (a) Show that the equation [2]

$$1 + \cos(x) = 2x^2 - 1$$

can be rearranged into the form

$$x = \pm\sqrt{a + b\cos(x)},$$

and state the values of a and b.

(b) Use the iteration formula [3]

$$x_{n+1} = \pm\sqrt{a + b\cos(x_n)},$$

with your values of a and b and with $x_0 = 1$ to find a root of the equation correct to 2 decimal places.

(c) Without further calculation write down another root of the equation and explain your answer. [2]

Total: 7

3. (a) Find the coordinates of the points where the curve $y = 4 - x^2$ crosses the x-axis. [2]

(b) The region bounded by the curve $y = 4 - x^2$ and the x-axis is rotated through 360° about the x-axis. Show that the volume of the solid generated is $\dfrac{512}{15}\pi$. [6]

Total: 8

4. A bicycle tyre develops a slow puncture.

The pressure, P pounds per square inch, in the tyre t minutes after the puncture occurs is given by

$$P = 14 + 50\mathrm{e}^{-kt}.$$

(a) Find the pressure in the tyre when the puncture occurs. [2]

Given that the pressure in the tyre is halved during the first 5 minutes after the puncture occurs, find correct to 3 significant figures

(b) the value of the constant k, [4]

(c) the pressure in the tyre 12 minutes after the puncture occurs. [2]

Total: 8

5. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}\colon x &\mapsto 3x^2 - 1, && x \in \mathbb{R}, \\ \mathrm{g}\colon x &\mapsto \mathrm{e}^{3x}, && x \in \mathbb{R}. \end{aligned}$$

   (a) Solve the equation $\mathrm{f}(x) = 26$. [3]

   (b) Evaluate $\mathrm{gf}(0.8)$ correct to 3 significant figures. [2]

   (c) Define $\mathrm{fg}(x)$ as simply as possible. [3]

Total: 8

6. (a) Simplify [5]

      i. $\dfrac{x^2 + 3x}{x^2 + 5x + 6}$,

      ii. $\dfrac{2x^2 - x - 1}{x^2 + 8x - 9}$.

   (b) Hence solve the equation [5]

$$\frac{x^2 + 3x}{x^2 + 5x + 6} = \frac{2x^2 - x - 1}{x^2 + 8x - 9},$$

   giving your answers in the form $a + b\sqrt{2}$.

Total: 10

7. (a) Prove that there are no real values of θ for which [4]

$$\cos(2\theta) + \cos(\theta) + 2 = 0.$$

   (b) Find the values of x in the interval $0 \le x \le 360°$, for which [5]

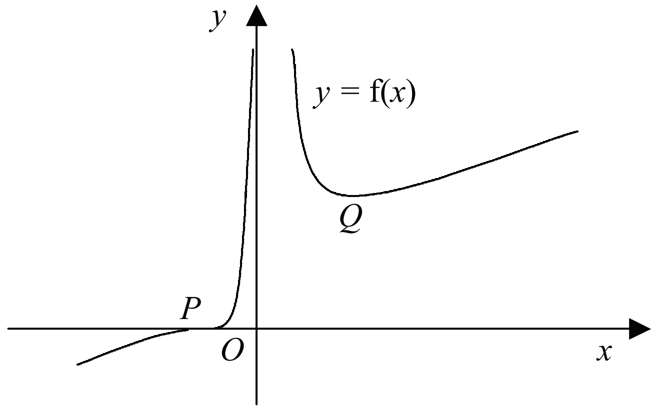
$$3\sin(x) - 2\cos^2(x) = 0.$$

   (c) Hence, find the values of y in the interval $0 \le y \le 180°$, for which [4]

$$3\sec(2y) - 2\cot(2y) = 0.$$

Total: 13

8. Figure shows the curve $y = \mathrm{f}(x)$ where $\mathrm{f}(x) \equiv \dfrac{(2+x)^3}{x^2}$.



(a) Express $(2+x)^3$ as a series in ascending powers of x. [2]

(b) Hence, express $\mathrm{f}(x)$ in the form $Ax^{-2}+Bx^{-1}+C+Dx$. [2]

The curve intersects the x-axis at the point P.

(c) Find the coordinates of P. [2]

(d) Show that $\mathrm{f}(x)$ is stationary at P. [4]

(e) Hence, find the coordinates of the other stationary point on the curve, Q. [6]

Total: 16