Solomon Practice Paper

Mechanics 3B

Time allowed: 90 **mintues**

Question	Points	Score
1	7	
2	8	
3	8	
4	12	
5	13	
6	13	
7	14	
Total:	75	

Centre:

Name:

Teacher:

1. A student is attempting to model the expansion of an airbag in a car following a collision.

   The student considers the displacement from the steering column, s metres, of a point P on the airbag t seconds after a collision and uses the formula

   $$s = \mathrm{e}^{3t} - 1, \qquad 0 \leq t \leq 0.1$$

   Using this model,

   (a) find, correct to the nearest centimetre, the maximum displacement of P, [2]

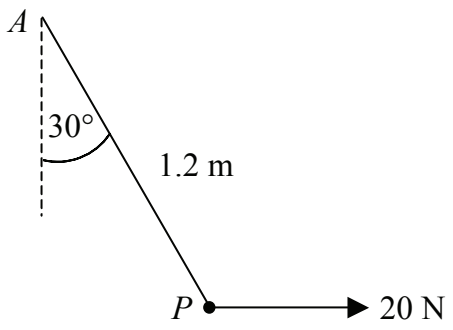
   (b) find the initial velocity of P, [3]

   (c) find the acceleration of P in terms of t. [1]

   (d) Explain why this model is unlikely to be realistic. [1]

   Total: 7

2. A particle P is attached to one end of a light elastic string of modulus of elasticity 80 N. The other end of the string is attached to a fixed point A.



   When a horizontal force of magnitude 20N is applied to P, it rests in equilibrium with the string making an angle of 30° with the vertical and $AP = 1.2$m as shown in Figure.

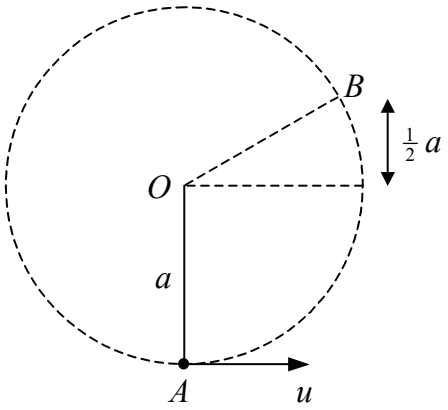
   (a) Find the tension in the string. [3]

   (b) Find the elastic potential energy stored in the string. [5]

   Total: 8

3. A particle of mass m is suspended at a point A vertically below a fixed point O by a light inextensible string of length a as shown in Figure.

   The particle is given a horizontal velocity u and subsequently moves along a circular arc until it reaches the point B where the string becomes slack.



Given that the point B is at a height $\frac{1}{2}a$ above the level of O,

(a) show that $\angle BOA = 120°$, [2]

(b) show that $u^2 = \frac{7}{2}ga$. [6]

Total: 8

4. On a particular day, high tide at the entrance to a harbour occurs at 11a.m. and the water depth is 14m. Low tide occurs $6\frac{1}{4}$ hours later at which time the water depth is 6m.

In a model of the situation, the water level is assumed to perform simple harmonic motion.

Using this model,

(a) write down the amplitude and period of the motion. [2]

A ship needs a depth of 9m before it can enter or leave the harbour.

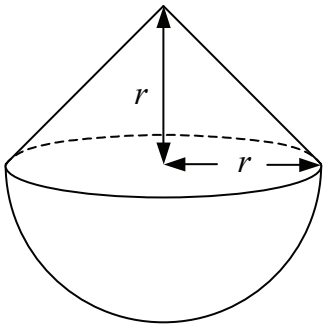
(b) Show that on this day a ship must enter the harbour by 2.38p.m., correct to the nearest minute, or wait for low tide to pass. [6]

Given that a ship is not ready to enter the harbour until 5 p.m.,

(c) find, to the nearest minute, how long the ship must wait before it can enter the harbour. [4]

Total: 12

5. (a) Use integration to show that the centre of mass of a uniform solid right circular cone of height h is 43h from the vertex of the cone. [6]



A paperweight is made by removing material from the top half of a solid sphere of radius r so that the remaining solid consists of a hemisphere of radius r and a cone of height r and base radius r as shown in Figure.

(b) Find the distance of the centre of mass of the paperweight from its vertex. [7]

Total: 13

6. A car is travelling on a horizontal racetrack round a circular bend of radius 40m. The coefficient of friction between the car and the road is $\frac{2}{5}$.

(a) Find the maximum speed at which the car can travel round the bend without slipping, giving your answer correct to 3 significant figures. [5]

The owner of the track decides to bank the corner at an angle of 25° in order to enable the cars to travel more quickly.

(b) Show that this increases the maximum speed at which the car can travel round the bend without slipping by 63%, correct to the nearest whole number. [8]

Total: 13

7. A particle is travelling along the x-axis. At time $t = 0$, the particle is at O and it travels such that its velocity, v ms^{-1}, at a distance x metres from O is given by

$$v = \frac{2}{x+1}.$$

The acceleration of the particle is a ms^{-2}.

(a) Show that $a = \dfrac{-4}{(x+1)^3}$. [4]

The points A and B lie on the x-axis. Given that the particle travels d metres from O to A in T seconds and 4 metres from A to B in 9 seconds,

(b) show that $d = 1.5$, [8]

(c) find T. [2]

Total: 14