Solomon Practice Paper

Mechanics 1J

Time allowed: 90 **mintues**

Question	Points	Score
1	8	
2	8	
3	10	
4	10	
5	11	
6	14	
7	14	
Total:	75	

Centre:

Name:

Teacher:

1. At time $t = 0$, a particle of mass 2kg has velocity $(8\mathbf{i} + \lambda\mathbf{j})$ ms^{-1} where $\mathbf{i}$ and $\mathbf{j}$ are horizontal perpendicular unit vectors and $\lambda > 0$.

   Given that the speed of the particle at time $t = 0$ is 17 ms^{-1},

   (a) find the value of λ. [3]

   The particle experiences a constant retarding force F so that when $t = 5$, it has velocity $(3\mathbf{i}+5\mathbf{j})$ ms^{-1}.

   (b) Show that F can be written in the form $\mu(\mathbf{i}+2\mathbf{j})$N where μ is a constant which you should find. [5]

   Total: 8

2. A monk uses a small brush to clean the stone floor of a monastery by pushing the brush with a force of P Newtons at an angle of 60° to the vertical. He moves the brush at a constant speed. The mass of the brush is 0.5kg and the coefficient of friction between the brush and the floor is $\frac{1}{\sqrt{3}}$. The brush is modelled as a particle and air resistance is ignored.

   (a) Show that $P = \frac{g}{2}$ Newtons. [7]

   (b) Explain why it is reasonable to ignore air resistance in this situation. [1]

   Total: 8

3. A small van of mass 1500kg is used to tow a car of mass 750kg by means of a rope of length 9m joined to both vehicles. The van sets off with the rope slack and reaches a speed of 2 ms^{-1} just before the rope becomes taut and jerks the car into motion. Immediately after the rope becomes taut, the van and car travel with common speed V ms^{-1}.

   (a) Show that $V = \frac{4}{3}$. [3]

   (b) Calculate the magnitude of the impulse on the car when the rope tightens. [2]

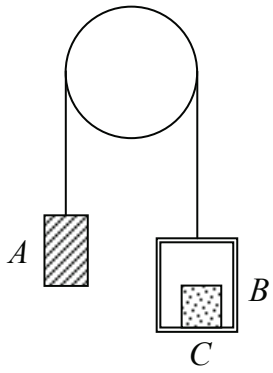
   The van and car eventually reach a steady speed of 18ms^{-1} with the rope taut when a child runs out into the road, 30m in front of the van. The van driver brakes sharply and decelerates uniformly to rest in a distance of 27m.

   It takes the driver of the car 1 second to react to the van starting to brake. He then brakes and the car decelerates uniformly at f ms^{-2}, coming to rest before colliding with the van.

   (c) Find the set of possible values of f. [5]

   Total: 10

4. Figure shows a weight A of mass 6kg connected by a light, inextensible string which passes over a smooth, fixed pulley to a box B of mass 5kg.



There is an object C of mass 3kg resting on the horizontal floor of box B.

The system is released from rest. Find, giving your answers in terms of g,

(a) the acceleration of the system, [4]

(b) the force on the pulley. [3]

(c) Show that the reaction between C and the floor of B is $\frac{18}{7}g$ newtons. [3]

Total: 10

5. Two flies P and Q, are crawling vertically up a wall. At time $t = 0$, the flies are at the same height above the ground, with P crawling at a steady speed of 4cms^{-1}.

Q starts from rest at time $t = 0$ and accelerates uniformly to a speed of 6cms^{-1} in 6 seconds. Fly Q then maintains this speed.

(a) Find the value of t when the two flies are moving at the same speed. [3]

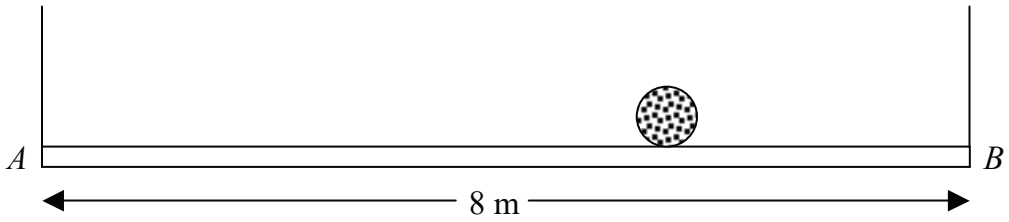
(b) Sketch on the same diagram, speed-time graphs to illustrate the motion of the two flies. [3]

Given that the distance of the two flies from the top of the wall at time $t = 0$ is x cm and that Q reaches the top of the wall first,

(c) show that $x > 36$. [5]

Total: 11

6. Figure shows a uniform plank AB of length 8 m and mass 50kg suspended horizontally by two light vertical inextensible strings attached at either end of the plank.



The maximum tension that either string can support is $40g$N.

A rock of mass Mkg is placed on the plank at A and rolled along the plank to B without either string breaking.

(a) Explain, with the aid of a sketch-graph, how the tension in the string at A varies with x, the distance of the rock from A. [3]

(b) Show that $M \leq 15$. [5]

The first rock is removed and a second rock of mass 20kg is placed on the plank.

(c) Find the fraction of the plank on which the rock can be placed without one of the strings breaking. [6]

Total: 14

7. At 6 a.m. a cargo ship has position vector $(7\mathbf{i} + 56\mathbf{j})$km relative to a fixed origin O on the coast and moves with constant velocity $(9\mathbf{i} - 6\mathbf{j})\text{kmh}^{-1}$.

A ferry sails from O at 6 a.m. and moves with constant velocity $(12\mathbf{i} + 18\mathbf{j})\text{kmh}^{-1}$. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed due east and due north respectively.

(a) Show that the position vector of the cargo ship t hours after 6 a.m. is given by $[(7 + 9t)\mathbf{i} + (56 - 6t)\mathbf{j}]$ km, and find the position vector of the ferry in terms of t. [3]

(b) Show that if both vessels maintain their course and speed, they will collide and find the time and position vector at which this occurs. [6]

At 8 a.m. the captain of the ferry realises that a collision is imminent and changes course so that the ferry now has velocity $(21\mathbf{i} + 6\mathbf{j})$ kmh^{-1}.

(c) Find the distance between the two ships at the time when they would have collided. [5]

Total: 14