Solomon Practice Paper

Further Pure Mathematics 3G

Time allowed: 90 **minutes**

Question	Points	Score
1	3	
2	6	
3	7	
4	9	
5	10	
6	10	
7	13	
8	17	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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1. [3]

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -4 \\ 1 & 2 & -1 \\ 2 & k & 0 \end{pmatrix}.$$

Find the value of the constant k for which $\mathbf{A}$ is a singular matrix.

2. Solve the equation [6]

$$z^3 = -4 + 4\sqrt{3}\mathbf{i},$$

giving your answers in the form $r(\cos\theta + \mathbf{i}\sin\theta)$ where $r > 0$ and $0 \leq \theta < 2\pi$.

3. Prove by induction that $n(n^2 + 5)$ is divisible by 6 for all positive integers n. [7]

4. The point P represents the complex number z in an Argand diagram.
   Given that

$$|z - 1 + 2\mathbf{i}| = 3,$$

   (a) sketch the locus of P in an Argand diagram. [3]

   T, U and V are transformations from the z–plane to the w–plane where

$$\begin{aligned} T: &\quad w = \ 4z \\ U: &\quad w = \ z + 5 - \mathbf{i} \\ V: &\quad w = \ z\mathrm{e}^{\mathbf{i}\frac{\pi}{2}} \end{aligned}$$

   (b) Describe exactly the locus of the image of P under each of these transformations. [6]

Total: 9

5. (a) By finding the first four derivatives of $f(x) = \cos(x)$, find the Taylor series expansion of $f(x)$ in ascending powers of $\left(x - \frac{\pi}{6}\right)$ up to and including the term in $\left(x - \frac{\pi}{6}\right)^3$. [5]

(b) Use this expansion to find an estimate of $\cos\left(\frac{\pi}{4}\right)$, giving your answer to 4 decimal places. [3]

(c) Find the percentage error in your answer to part (b), giving your answer to 2 significant figures. [2]

Total: 10

6. Given that y satisfies the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x^2 + xy - y^2, \qquad y = \frac{1}{2} \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = -1 \qquad \text{at} \quad x = 0.$$

(a) use the Taylor series method to obtain a series for y in ascending powers of x up to and including the term in x^3. [6]

(b) Use your series to estimate the value of y at $x = -0.1$. [1]

(c) Use the approximation [3]

$$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$$

with a step length of 0.1 and your answer to part (b) to estimate the value of y when $x = 0.1$.

Total: 10

7. Referred to a fixed origin, the straight lines l_1, l_2 and l_3 have equations

$$\begin{aligned} l_1: \quad \mathbf{r} &= 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 4\mathbf{j} + \mathbf{k}) \\ l_2: \quad \mathbf{r} &= 3\mathbf{i} + 4\mathbf{k} + t(4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) \\ l_3: \quad \mathbf{r} &= \mathbf{i} - 2\mathbf{j} + u(2\mathbf{j} + \mathbf{k}) \end{aligned}$$

The acute angle between l_1 and l_2 is θ.

(a) Find the exact value of $\sin\theta$. [5]

The plane Π contains the lines l_1 and l_2.

(b) Find an equation of Π, giving your answer in the form $ax + by + cz + d = 0$. [4]

(c) Show that the line l_3 lies on the plane Π. [4]

Total: 13

8. (a) $\mathbf{A}$ and $\mathbf{B}$ are non–singular square matrices. Prove that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$. [4]

The transformations $S : \mathbb{R}^2 \to \mathbb{R}^2$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$ are defined by

$$S : \quad \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} y - x \\ 2x + y \end{pmatrix} \quad \text{and} \quad T : \quad \begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 3x \\ x + y \end{pmatrix}.$$

(b) Show that S represents a linear transformation. [7]

(c) Using your result in (a), or otherwise, find the matrix that represents the transformation $(ST)^{-1}$. [6]

Total: 17