

# Solomon Practice Paper

## Further Pure Mathematics 3G

Time allowed: 90 minutes

Centre: [www.CasperYC.club](http://www.CasperYC.club)

Name:

Teacher:

Question	Points	Score
1	3	
2	6	
3	7	
4	9	
5	10	
6	10	
7	13	
8	17	
Total:	75	

How I can achieve better:

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Last updated:

July 14, 2025

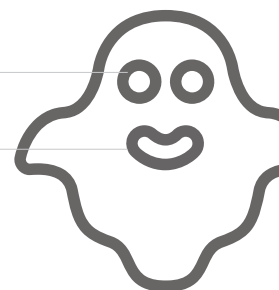


1.

[3]

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & -4 \\ 1 & 2 & -1 \\ 2 & k & 0 \end{pmatrix}.$$

Find the value of the constant  $k$  for which  $\mathbf{A}$  is a singular matrix.

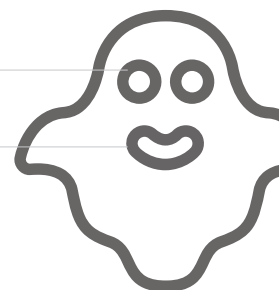


2. Solve the equation

[6]

$$z^3 = -4 + 4\sqrt{3}\mathbf{i},$$

giving your answers in the form  $r(\cos \theta + \mathbf{i} \sin \theta)$  where  $r > 0$  and  $0 \leq \theta < 2\pi$ .



3. Prove by induction that  $n(n^2 + 5)$  is divisible by 6 for all positive integers  $n$ .

[7]



4. The point  $P$  represents the complex number  $z$  in an Argand diagram.

Given that

$$|z - 1 + 2\mathbf{i}| = 3,$$

- (a) sketch the locus of  $P$  in an Argand diagram.

[3]

$T, U$  and  $V$  are transformations from the  $z$ -plane to the  $w$ -plane where

$$T : \quad w = 4z$$

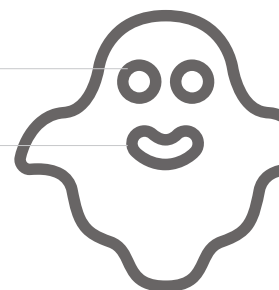
$$U: \quad w = z + 5 - \mathbf{i}$$

$$V: \quad w = ze^{i\frac{\pi}{2}}$$

- (b) Describe exactly the locus of the image of  $P$  under each of these transformations.

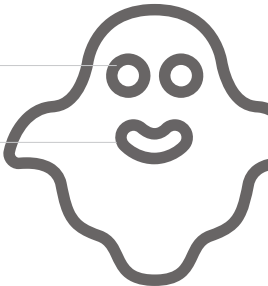
[6]

Total: 9



5. (a) By finding the first four derivatives of  $f(x) = \cos(x)$ , find the Taylor series expansion of  $f(x)$  in ascending powers of  $\left(x - \frac{\pi}{6}\right)$  up to and including the term in  $\left(x - \frac{\pi}{6}\right)^3$ . [5]
- (b) Use this expansion to find an estimate of  $\cos\left(\frac{\pi}{4}\right)$ , giving your answer to 4 decimal places. [3]
- (c) Find the percentage error in your answer to part (b), giving your answer to 2 significant figures. [2]

Total: 10



$$\frac{d^2y}{dx^2} = x^2 + xy - y^2, \quad y = \frac{1}{2} \quad \text{and} \quad \frac{dy}{dx} = -1 \quad \text{at} \quad x = 0.$$

- (c) Use the approximation  $\ln(1+x) \approx x - \frac{x^2}{2} + \frac{x^3}{3}$  for  $x \ll 1$ . [3]

$$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$$

Total: 10



7. Referred to a fixed origin, the straight lines  $l_1, l_2$  and  $l_3$  have equations

$$l_1 : \quad \mathbf{r} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k} + s(2\mathbf{i} - 4\mathbf{j} + \mathbf{k})$$

$$l_2: \quad \mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + t(4\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})$$

$$l_3 : \quad \mathbf{r} = \mathbf{i} - 2\mathbf{j} + u(2\mathbf{j} + \mathbf{k})$$

The acute angle between  $l_1$  and  $l_2$  is  $\theta$ .

(a) Find the exact value of  $\sin \theta$ .

[5]

The plane  $\Pi$  contains the lines  $l_1$  and  $l_2$ .

(b) Find an equation of  $\Pi$ , giving your answer in the form  $ax + by + cz + d = 0$ .

[4]

(c) Show that the line  $l_3$  lies on the plane  $\Pi$ .

[4]

Total: 13





8. (a) **A** and **B** are non-singular square matrices. Prove that  $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ . [4]

The transformations  $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  and  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  are defined by

$$S : \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} y - x \\ 2x + y \end{pmatrix} \quad \text{and} \quad T : \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} 3x \\ x + y \end{pmatrix}.$$

- (b) Show that  $S$  represents a linear transformation. [7]

- (c) Using your result in (a), or otherwise, find the matrix that represents the transformation  $(ST)^{-1}$ . [6]

Total: 17

