

Solomon Practice Paper

Further Pure Mathematics 3F

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	11	
4	11	
5	11	
6	13	
7	15	
Total:	75	

How I can achieve better:

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Last updated:

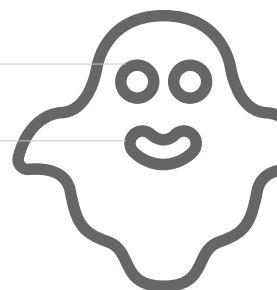
July 14, 2025



1. Prove by induction that, for all $n \in \mathbb{Z}^+$,

[6]

$$\sum_{r=1}^n \ln \frac{r+1}{r} = \ln(n+1).$$



2.

$$\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 3 & -6 \end{pmatrix}.$$

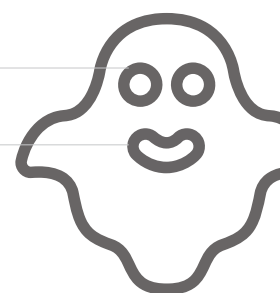
(a) Find the eigenvalues of \mathbf{M} .

[4]

(b) Find eigenvectors corresponding to each eigenvalue found in part (a).

[4]

Total: 8



3. A transformation T from the z -plane to the w -plane is defined by

$$w = \frac{z + 2\mathbf{i}}{z - \mathbf{i}}, \quad z \neq \mathbf{i},$$

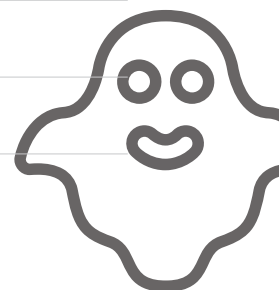
where $z = x + \mathbf{i}y$, $w = u + \mathbf{i}v$ and x, y, u and v are real.

- (a) Show that the circle $|z| = 1$ is mapped onto a straight line in the w -plane under T and find an equation of the line. [5]

The circle $|z - (a + \mathbf{i}b)| = r$ in the z -plane is mapped under T onto the circle $|w| = 2$ in the w -plane, where a, b and r are real.

- (b) Find the values of a, b and r . [6]

Total: 11



4. The points A, B and C with coordinates $(x_{-1}, y_{-1}), (x_0, y_0)$ and (x_1, y_1) respectively lie on the curve $y = f(x)$ with $x_1 - x_0 = x_0 - x_{-1} = h$.

- (a) Use the first three terms of the Taylor series expansion in ascending powers of $(x - x_0)$ to show that [5]

$$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}.$$

The variable y satisfies the differential equation

$$\frac{d^2y}{dx^2} + (x + 2)\frac{dy}{dx} - 3y = 0 \quad \text{with } y = 1 \text{ at } x = 0 \text{ and } y = 1.2 \text{ at } x = 0.1.$$

- (b) Use the approximations [6]

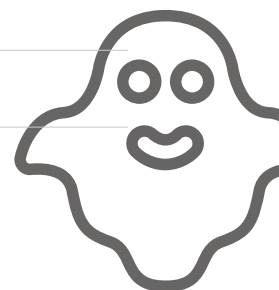
$$\left(\frac{dy}{dx}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$$

and

$$\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$$

with a step length of 0.1 to estimate the value of y at $x = 0.2$.

Total: 11



5.

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 3 \\ 4 & q & 1 \\ 1 & 2 & -1 \end{pmatrix}, \quad q \neq 4\frac{1}{4}.$$

(a) Find \mathbf{A}^{-1} in terms of q .

[7]

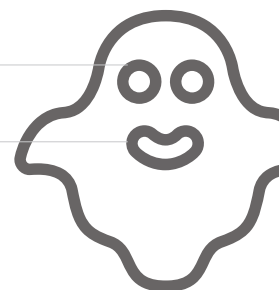
(b) Hence, or otherwise, solve the simultaneous equations

[4]

$$\begin{array}{rclcl} x & - & y & + & 3z & = & 1 \\ 4x & + & y & + & z & = & 2 \\ x & + & 2y & - & z & = & 5 \end{array}$$

showing your working clearly.

Total: 11



6. Given that

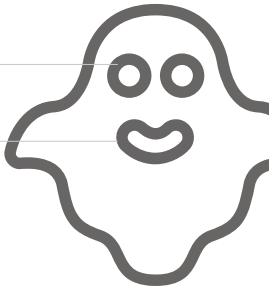
$$y = \sqrt{1 - x^2} \cos^{-1}(x),$$

(a) show that [5]

$$(1 - x^2) \frac{dy}{dx} + xy - x^2 + 1 = 0. \tag{*}$$

(b) By differentiating equation (*) twice, or otherwise, obtain the Maclaurin expansion of $y = \sqrt{1 - x^2} \cos^{-1}(x)$ up to and including the term in x^3 . [8]

Total: 13



7. The plane Π_1 has vector equation

$$\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + \mathbf{k}).$$

- (a) Find a vector \mathbf{n} which is normal to Π_1 . [3]
- (b) Hence find a vector equation of Π_1 in the form $\mathbf{r} \cdot \mathbf{n} = p$. [2]
- (c) Find the perpendicular distance between Π_1 and the point A with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, [4]
giving your answer in the form $a\sqrt{6}$, where $a \in \mathbb{Q}$.

The plane Π_2 has equation $\mathbf{r} \cdot (\mathbf{i} + b\mathbf{j}) = -4$.

The angle between Π_1 and Π_2 is 30° .

- (d) Find the possible values of the constant b . [6]

Total: 15

