Solomon Practice Paper

Further Pure Mathematics 3A

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	6	
3	7	
4	9	
5	11	
6	11	
7	11	
8	14	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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1. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$\begin{aligned} l_1 : \quad & [\mathbf{r} - (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})] \times (\mathbf{i} + \mathbf{k}) = 0, \\ l_2 : \quad & [\mathbf{r} - (\mathbf{i} + \mathbf{j} + 4\mathbf{k})] \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) = 0. \end{aligned}$$

(a) Find $(\mathbf{i} + \mathbf{k}) \times (2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$. [3]

(b) Find the shortest distance between l_1 and l_2. [3]

Total: 6

2. Prove by induction that, for all $n \in \mathbb{Z}^+$, [6]

$$\sum_{r=1}^{n}(r^2+1)r! = n(n+1)!$$

3. (a) Solve the equation [5]

$$z^3 + 27 = 0,$$

giving your answers in the form $re^{i\theta}$ where $r > 0$, $-\pi < \theta \leq \pi$.

(b) Show the points representing your solutions on an Argand diagram. [2]

Total: 7

4.

$$\mathbf{A} = \begin{pmatrix} 2 & a \\ 2 & b \end{pmatrix}.$$

The matrix $\mathbf{A}$ has eigenvalues $\lambda_1 = -2$ and $\lambda_2 = 3$.

(a) Find the value of a and the value of b. [4]

Using your values of a and b,

(b) for each eigenvalue, find a corresponding eigenvector, [3]

(c) find a matrix $\mathbf{P}$ such that [2]

$$\mathbf{P}^{\mathbf{T}}\mathbf{A}\mathbf{P} = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}.$$

Total: 9

5. [11]

$$\left(1+x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=0$$

and

$$y=1 \quad \text{and} \quad \frac{\mathrm{d}y}{\mathrm{d}x}=1 \quad \text{at} \quad x=-1.$$

Find a series solution of the differential equation in ascending powers of $(x+1)$ up to and including the term in $(x+1)^4$.

6. The variable y satisfies the differential equation [11]

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x\frac{\mathrm{d}y}{\mathrm{d}x} + y^2$$

and

$$y = 1.2 \quad \text{at} \quad x = 0.1 \quad \text{and} \quad y = 0.9 \quad \text{at} \quad x = 0.2.$$

Use the approximations

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \frac{y_1 - y_{-1}}{2h}$$

and

$$\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$$

with a step length of 0.1 to estimate the values of y at $x = 0.3$ and $x = 0.4$ giving your answers to 3 significant figures.

7.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 1 \\ k & 4 & 3 \\ -1 & k & 2 \end{pmatrix}.$$

(a) Find the determinant of $\mathbf{M}$ in terms of k. [2]

(b) Prove that $\mathbf{M}$ is non–singular for all real values of k. [2]

(c) Given that $k = 3$, find $\mathbf{M^{-1}}$, showing each step of your working. [4]

When $k = 3$ the image of the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ when transformed by $\mathbf{M}$ is the vector $\begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}$.

(d) Find the values of a, b and c. [3]

Total: 11

8. A transformation T from the z–plane to the w–plane is defined by

$$w = \frac{z+1}{\mathbf{i}z-1}, \qquad z \neq -\mathbf{i},$$

where $z = x + \mathbf{i}y, w = u + \mathbf{i}v$ and x, y, u and v are real.
T transforms the circle $|z| = 1$ in the z–plane onto a straight line L in the w–plane.

(a) Find an equation of L giving your answer in terms of u and v. [5]

(b) Show that T transforms the line $\text{Im}(z) = 0$ in the z–plane onto a circle C in the w–plane, giving the centre and radius of this circle. [6]

(c) On a single Argand diagram sketch L and C. [3]

Total: 14