Solomon Practice Paper

Further Pure Mathematics 2G

Time allowed: 90 **minutes**

Question	Points	Score
1	7	
2	7	
3	8	
4	9	
5	12	
6	14	
7	18	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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Last updated: July 14, 2025

1. Given that $y = e^{\tan^{-1}(x)}$,

   (a) find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [4]

   The curve $y = e^{\tan^{-1}(x)}$ has a point of inflexion.

   (b) Find the coordinates of this point of inflexion. [3]

Total: 7

2. (a) Prove that [3]

$$\frac{\mathrm{d}}{\mathrm{d}x}\cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}}.$$

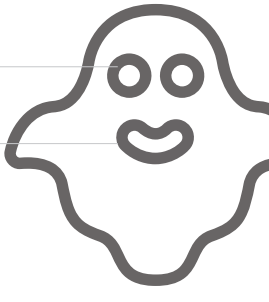
(b) Find [4]

$$\int \cosh^{-1}(x)\,\mathrm{d}x.$$

Total: 7

3. Find [8]

$$\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin(2x)}\,\mathrm{d}x.$$



4. (a) Find [6]

$$\int \frac{1}{\sqrt{4x^2 - 4x + 10}}\,\mathrm{d}x.$$

(b) Hence evaluate [3]

$$\int_{\frac{1}{2}}^{2} \frac{1}{\sqrt{4x^2 - 4x + 10}}\,\mathrm{d}x.$$

giving your answer in terms of natural logarithms.

Total: 9

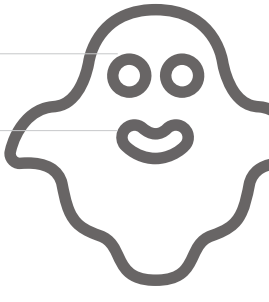
5. (a) On the same axes sketch the curves with equations $y = 2 - \tanh(x)$ and $y = 3\,\mathrm{sech}(x)$, giving the coordinates of the points of intersection of the curves with the coordinate axes and the equations of the asymptotes. [5]

   (b) Solve the equation [7]

$$2 - \tanh(x) = 3\,\mathrm{sech}(x),$$

   giving your answers to 2 decimal places.

Total: 12



6.

$$I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,\mathrm{d}x, \qquad n \geq 0.$$

(a) Show that [7]

$$I_n = \frac{n-1}{n} I_{n-2}, \qquad n \geq 2.$$

The curve C is defined by $y = \sin^2(x)$, $0 \leq x \leq \pi$.
The area bounded by C and the positive x–axis is rotated through 2π radians about the x–axis.

(b) Find the volume of the solid generated giving your answer in terms of π. [7]

Total: 14

7.

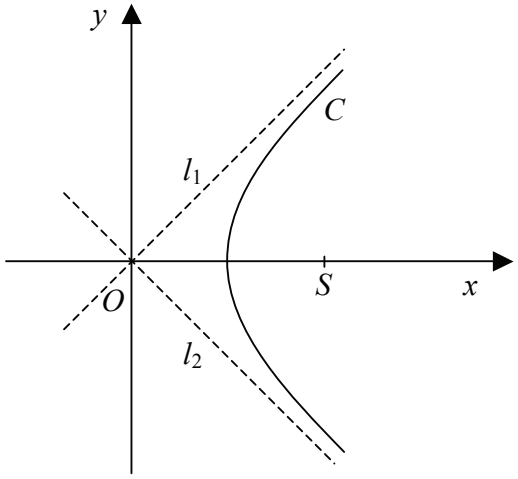


Figure above shows the curve C which is part of the hyperbola with parametric equations

$$x = a\cosh(t) \quad \text{and} \quad y = 2a\sinh(t),$$

where a is a positive constant and $x \geq a$. The lines l_1 and l_2 are asymptotes to C.

(a) Show that the radius of curvature of C at its vertex is $4a$. [6]

(b) Show that an equation of the tangent to C at the point $P(a\cosh(p), 2a\sinh(p))$ is [4]

$$2x\cosh(p) - y\sinh(p) = 2a.$$

The tangent to the curve C at P meets the asymptote l_1 at Q.
Given that QS is parallel to the y–axis, where S is the focus,

(c) show that $p = \frac{1}{2}\ln(5)$. [8]

Total: 18