Solomon Practice Paper

Further Pure Mathematics 2F

Time allowed: 90 **minutes**

Question	Points	Score
1	4	
2	7	
3	7	
4	9	
5	10	
6	11	
7	12	
8	15	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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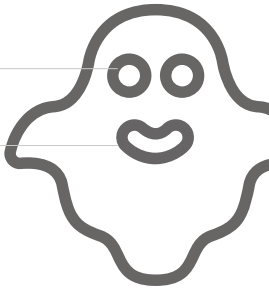
Last updated: July 14, 2025

1. [4]

$$\mathrm{f}(x) = \tanh^{-1}(\sin(x)).$$

Show that $\mathrm{f}'(x) = \sec(x)$.

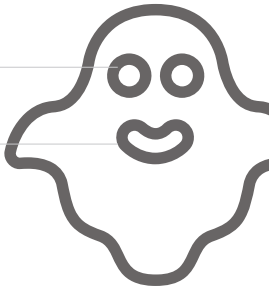
2. Find the length of the arc of the curve with equation $y = \ln(\sec x)$ between $x = 0$ and $x = \frac{\pi}{3}$, giving your answer in terms of natural logarithms. [7]



3. A curve has parametric equations [7]

$$x = t^2 \quad \text{and} \quad y = t^3,$$

Show that the radius of curvature of the curve at the point $(1, 1)$ is $\dfrac{13\sqrt{3}}{6}$.



4.

$$I_n = \int_1^{\mathrm{e}} \Big[\ln(x)\Big]^n \,\mathrm{d}x.$$

(a) Prove that, for $n \in \mathbb{Z}^+$, [4]

$$I_n = \mathrm{e} - nI_{n-1}.$$

(b) Find I_3, leaving your answer in terms of e. [5]

Total: 9

5. [10]

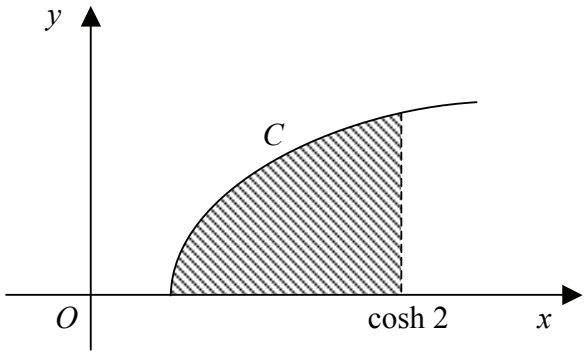


Figure above shows the curve C which has equation $y = \cosh^{-1}(x)$.

The shaded region bounded by C, the x–axis and the line $x = \cosh 2$ is rotated through 2π about the y–axis.

The volume of revolution of the solid generated is $a\pi$.

Find the value of a to one decimal place.

6.

$$\mathrm{f}(x) \equiv \frac{3x-7}{(x+1)(x^2+4)}, \qquad x \neq -1.$$

(a) Express $\mathrm{f}(x)$ in partial fractions. [4]

(b) Show that [7]

$$\int_0^2 \mathrm{f}(x)\,\mathrm{d}x = \frac{\pi}{8} + \ln\left(\frac{2}{9}\right).$$

Total: 11

7. The ellipse C has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

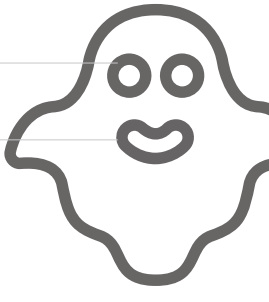
where a and b are positive constants and $a > b$.

(a) Find an equation of the normal to C at the point $P(a\cos\theta, b\sin\theta)$. [5]

The normal to C at P meets the x–axis at Q.
R is the foot of the perpendicular from P to the x–axis.

(b) Show that $\dfrac{OQ}{QR} = e^2$, where e is the eccentricity of C. [7]

Total: 12



8. (a) Using the definitions of hyperbolic functions in terms of exponential functions prove that [6]

$$\sinh^{-1}(x) = \ln\left|x + \sqrt{x^2+1}\right|.$$

(b) On the same axes sketch the graphs of $y = \sinh x$ and $y = \sinh^{-1}(x)$. [3]

(c) Solve the equation [6]

$$x = \sinh\left[\ln(3x-2)\right], \qquad x > \frac{2}{3}.$$

Total: 15