Solomon Practice Paper

Further Pure Mathematics 2E

Time allowed: 90 **minutes**

Question	Points	Score
1	4	
2	6	
3	9	
4	9	
5	11	
6	11	
7	11	
8	14	
Total:	75	

Centre: www.CasperYC.club

Name:

Teacher:

How I can achieve better:

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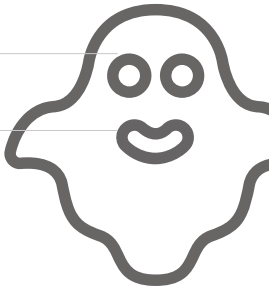
1. A student without a calculator must find the value of x given that [4]

$$\operatorname{arctanh}(x) = \ln(3).$$

With clear working, show how the student could find x and state the value he should obtain.



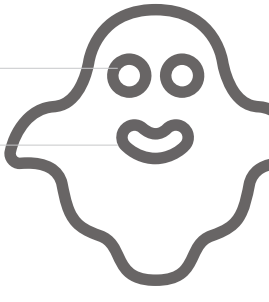
2.

$$\mathrm{f}(x) = \sin(2x) - x\cosh^2(x).$$

(a) Find $\mathrm{f}'(x)$. [3]

(b) Show that the curve with equation $y = \mathrm{f}(x)$ has a stationary point in the interval $0.3 < x < 0.4$. [3]

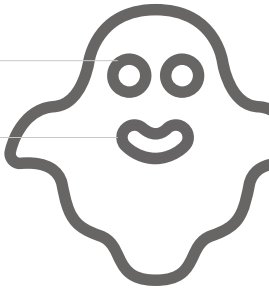
Total: 6



3. Given that [9]

$$\int_0^{\frac{2\pi}{3}} \frac{1}{5+4\cos(x)}\,\mathrm{d}x = a\pi, \qquad a \in \mathbb{Q},$$

use the substitution $t = \tan\left(\frac{1}{2}\right)$ to find the value of a.



4. The curve C has equation [9]

$$y = a\cosh\left(\frac{x}{a}\right),$$

where a is a positive constant.

The area bounded by the curve C, the x–axis and the lines $x = -a$ and $x = a$ is rotated through 2π radians about the x–axis.

Show that the curved surface area of the solid generated is $\pi a^2(\sinh(2) + 2)$.

5. The intrinsic equation of the curve C is $s = 2\psi$.
   Given that s is measured from the origin,

   (a) find a Cartesian equation of C, [9]

   (b) sketch C. [2]

Total: 11

6. (a) Using the definitions of hyperbolic functions in terms of exponential functions, prove that [4]

$$\cosh(x+y) \equiv \cosh(x)\cosh(y) + \sinh(x)\sinh(y).$$

Given that

$$5\cosh(x) + 4\sinh(x) \equiv R\cosh(x+a),$$

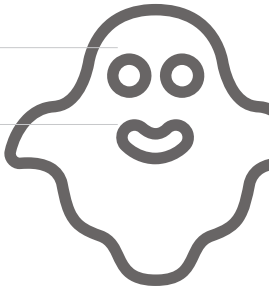
find

(b) the value of R, [3]

(c) the value of α, giving your answer in terms of natural logarithms. [3]

(d) Hence, or otherwise, state the minimum value of $5\cosh(x) + 4\sinh(x)$. [1]

Total: 11



7.

$$I_n = \int_0^1 x^n \mathrm{e}^{x^2}\,\mathrm{d}x, \qquad n \geq 0.$$

(a) Show that [5]

$$I_n = \frac{1}{2}\mathrm{e} = \frac{1}{2}(n-1)I_{n-2}, \qquad n \geq 2.$$

(b) Hence find [6]

$$I_n = \int_0^1 x^5 \mathrm{e}^{x^2}\,\mathrm{d}x,$$

giving your answer in terms of e.

Total: 11

8. The line with equation $y = mx + c$ is a tangent to the parabola with equation $y^2 = 8x$.

   (a) Show that $mc = 2$. [5]

   The lines l_1 and l_2 are tangents to both the parabola with equation $y^2 = 8x$ and the circle with equation $x^2 + y^2 = 2$.

   (b) Find the equations of l_1 and l_2. [9]

Total: 14