# Solomon Practice Paper

## Further Pure Mathematics 2D

**Time allowed:** 90 **minutes**

| Question | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 11 | |
| 4 | 12 | |
| 5 | 13 | |
| 6 | 13 | |
| 7 | 16 | |
| Total: | 75 | |

**Centre:** www.CasperYC.club

**Name:**

**Teacher:**

**How I can achieve better:**

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1.

$$y = \frac{\text{cosech}(x)}{x^2 + 1}$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [4]

(b) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0.5$, giving your answer to 2 decimal places. [1]

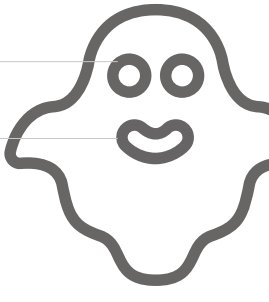
Total: 5

2. A curve has intrinsic coordinates $(s, \psi)$ and radius of curvature $\rho$. [5]

Given that $\rho = 2(s + a)$, where $a$ is constant, show that the intrinsic equation of the curve can be written in the form

$$s = A\mathrm{e}^{2\psi} - a,$$

where $A$ is constant.



3. (a) Prove that [5]

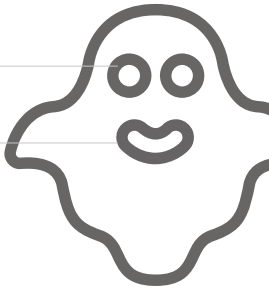
$$\sinh(3x) \equiv 4\sinh^3(x) + 3\sinh(x).$$

(b) Hence, or otherwise, solve the equation [6]

$$\sinh(3x) \equiv 7\sinh^2(x),$$

giving your answers in terms of natural logarithms where appropriate.

Total: 11



4. (a) Find [3]

$$\int \frac{1}{\sqrt{9-4x^2}}\,\mathrm{d}x.$$

(b) Find [3]

$$\int \frac{1-2x}{\sqrt{9-4x^2}}\,\mathrm{d}x.$$

(c) Hence, or otherwise, solve the differential equation [6]

$$\sqrt{9-4x^2}\frac{\mathrm{d}y}{\mathrm{d}x} = y(1-2x),$$

given that $y = 1$ when $x = 0$.

Total: 12

5. The curve $C$ has equation $y^2 = 4ax$, where a is $a$ positive constant.

   (a) Show that an equation of the tangent to $C$ at the point $P(ap^2, 2ap)$, $p \neq 0$, is [4]

   $$yp = x + ap^2.$$

   The point $Q(aq^2, 2aq)$, is on $C$ where $q \neq 0$ and $p \neq q$.
   The chord $PQ$ passes through the focus of $C$.

   Show that

   (b) $pq = -1$, [5]

   (c) the tangent to $C$ at $P$ and the tangent to $C$ at $Q$ meet on the directrix of $C$. [4]

Total: 13

6.

$$I_n = \int_0^{\frac{\pi}{4}} \sec^n(x)\,\mathrm{d}x, \qquad n \neq 0.$$

(a) Show that [7]

$$(n-1)I_n = \left(\sqrt{2}\right)^{n-2} + (n-2)I_{n-2}, \qquad n \geq 2.$$

(b) Hence find the exact value of $I_3$, giving your answer in terms of natural logarithms. [6]

Total: 13

7. (a) Show that [9]

$$\int \sqrt{a^2 + x^2}\,\mathrm{d}x = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\operatorname{arcsinh}\left(\frac{x}{a}\right) + c.$$

The parametric equations of the curve $C$ are

$$x = 2t \quad \text{and} \quad y = t^2, \qquad 0 \leq t \leq 3.$$

(b) Show that the length of $C$ is given by [4]

$$2\int_0^3 \sqrt{1 + t^2}\,\mathrm{d}t.$$

(c) Find the length of $C$. [3]

Total: 16