Solomon Practice Paper

Further Pure Mathematics 1A

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	6	
3	6	
4	7	
5	9	
6	12	
7	14	
8	15	
Total:	75	

Centre: www.CasperYC.club

Name:

Teacher:

How I can achieve better:

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Last updated: July 14, 2025

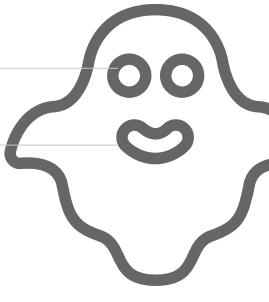
1.

$$\mathrm{f}(z) = z^3 - 5z^2 + 17z - 13.$$

(a) Show that $(z - 1)$ is a factor of $\mathrm{f}(z)$. [1]

(b) Hence find all the roots of the equation $\mathrm{f}(z) = 0$, giving your answers in the form $a + \mathbf{i}b$ where a and b are integers. [5]

Total: 6



2. Find the general solution of the differential equation [6]

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \frac{\mathrm{e}^x}{x^2},$$

giving your answer in the form $y = \mathrm{f}(x)$

3. (a) Express $\dfrac{1}{r(r+1)}$ in partial fractions. [2]

   (b) Hence, or otherwise, find [4]

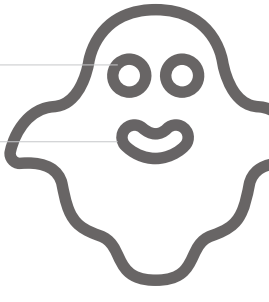
$$\sum_{r+3}^{35} \frac{1}{r(r+1)},$$

giving your answer as a fraction in its lowest terms.

Total: 6

4. Find the set of values of x for which [7]

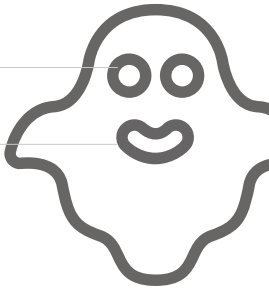
$$\frac{(x-3)^2}{x+1} < 2.$$



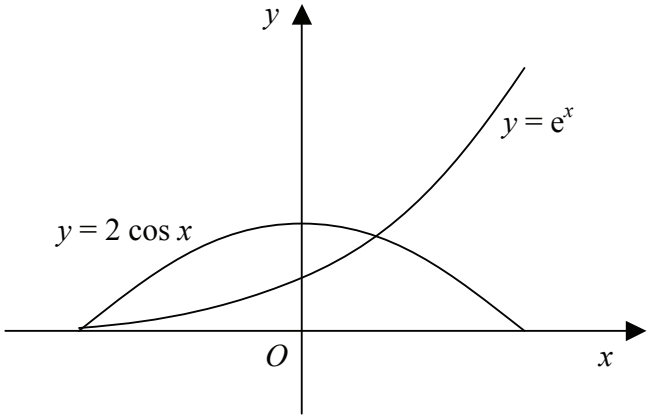
5. (a) Sketch the curve with polar equation $r = a\cos(3\theta), a > 0$, for $0 \leq \theta \leq \pi$. [3]

   (b) Show that the total area enclosed by the curve $r = a\cos(3\theta)$ is $\frac{\pi a^2}{4}$. [6]

Total: 9



6. Figure shows the curves $y = 2\cos x$ and $y = \mathrm{e}^x$ in the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.



Given that $\mathrm{f}(x) \equiv \mathrm{e}^x - 2\cos(x)$,

(a) Write down the number of solutions of the equation $\mathrm{f}(x) = 0$ in the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. [1]

(b) Show that the equation $\mathrm{f}(x) = 0$ has a solution, α, in the interval $[0, 1]$. [2]

(c) Using 0.5 as a first approximation to α, use the Newton–Raphson process once to find an improved estimate for α, giving your answer correct to 2 decimal places. [4]

(d) Show that the estimate of α obtained in part (c) is accurate to 2 decimal places. [2]

There is another root, β, of the equation $\mathrm{f}(x) = 0$ in the interval $[-2, -1]$.

(e) Use linear interpolation once on this interval to estimate the value of β, giving your answer correct to 2 decimal places. [3]

Total: 12

7. The complex numbers z and w are such that

$$z = \frac{A}{1 - \mathbf{i}} \quad \text{and} \quad w = \frac{B}{2 + \mathbf{i}}$$

where A and B are real.
Given that $z + w = 6$,

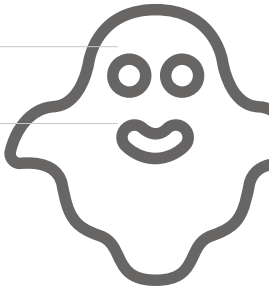
(a) find A and B. [6]

z and w are represented by the points P and Q respectively on an Argand diagram.

(b) Show P and Q on the same Argand diagram. [5]

(c) Find the distance PQ in the form $a\sqrt{5}$. [3]

Total: 14



8. (a) Find the values of p and q such that $x = p\cos(t) + q\sin(t)$ satisfies the differential equation [6]

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = \sin(t).$$

(b) Hence find the solution of this differential equation for which $x = 1$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2}$ at $t = 0$. [9]

Total: 15