

Solomon Practice Paper

Core Mathematics 4J

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

| Question | Points | Score |
|----------|--------|-------|
| 1 | 6 | |
| 2 | 6 | |
| 3 | 8 | |
| 4 | 9 | |
| 5 | 9 | |
| 6 | 10 | |
| 7 | 13 | |
| 8 | 14 | |
| Total: | 75 | |

How I can achieve better:

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Last updated: May 5, 2023



1. The region bounded by the curve $y = x^2 - 2x$ and the x -axis is rotated through 2π radians about the x -axis. [6]

Find the volume of the solid formed, giving your answer in terms of π .

2. Use the substitution $u = 1 - x^{\frac{1}{2}}$ to find [6]

$$\int \frac{1}{1 - x^{\frac{1}{2}}} dx.$$

3. A curve has the equation

$$2 \sin(2x) - \tan(y) = 0.$$

- (a) Find an expression for $\frac{dy}{dx}$ in its simplest form in terms of x and y . [5]

- (b) Show that the tangent to the curve at the point $(\frac{\pi}{6}, \frac{\pi}{3})$ has the equation [3]

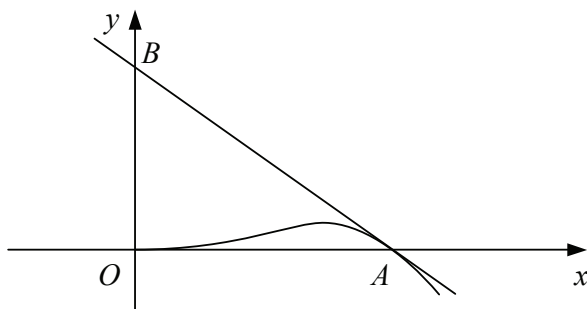
$$y = \frac{1}{2}x + \frac{\pi}{4}.$$

Total: 8

4. Figure shows the curve with parametric equations

$$x = a\sqrt{t}, \quad \text{and} \quad y = at(1 - t), \quad t \geq 0,$$

where a is a positive constant.



- (a) Find $\frac{dy}{dx}$ in terms of t . [3]

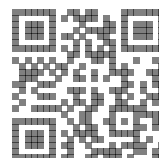
The curve meets the x -axis at the origin, O , and at the point A . The tangent to the curve at A meets the y -axis at the point B as shown.

- (b) Show that the area of triangle OAB is a^2 . [6]

Total: 9

5. The gradient at any point (x, y) on a curve is proportional to \sqrt{y} .

Given that the curve passes through the point with coordinates $(0, 4)$,



(a) show that the equation of the curve can be written in the form

[5]

$$2\sqrt{y} = kx + 4,$$

where k is a positive constant.

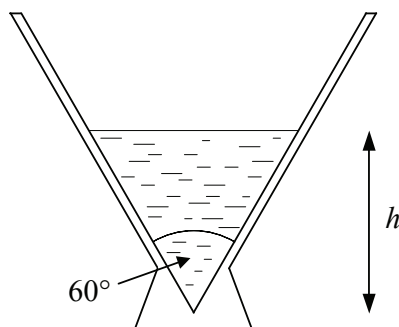
Given also that the curve passes through the point with coordinates $(2, 9)$,

(b) find the equation of the curve in the form $y = f(x)$.

[4]

Total: 9

6. Figure shows a vertical cross-section of a vase.



The inside of the vase is in the shape of a right-circular cone with the angle between the sides in the cross-section being 60° . When the depth of water in the vase is h cm, the volume of water in the vase is V cm³.

(a) Show that $V = \frac{1}{9}\pi h^3$.

[3]

The vase is initially empty and water is poured in at a constant rate of 120 cm³ s⁻¹.

(b) Find, to 2 decimal places, the rate at which h is increasing

[7]

i. when $h = 6$,

ii. after water has been poured in for 8 seconds.

Total: 10

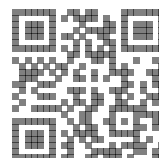
7. Relative to a fixed origin, the points A and B have position vectors

$$\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix}$$

respectively.

(a) Find a vector equation for the line l_1 which passes through A and B .

[2]



The line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -7 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

- (b) Show that lines l_1 and l_2 do not intersect. [5]
- (c) Find the position vector of the point C on l_2 such that $\angle ABC = 90^\circ$. [6]

Total: 13

8.

$$f(x) = \frac{x(3x - 7)}{(1 - x)(1 - 3x)}, \quad |x| < \frac{1}{3}.$$

- (a) Find the values of the constants A , B and C such that [4]

$$f(x) = A + \frac{B}{1 - x} + \frac{C}{1 - 3x}.$$

- (b) Evaluate [5]

$$\int_0^{\frac{1}{4}} f(x) \, dx,$$

giving your answer in the form $p + \ln(q)$, where p and q are rational.

- (c) Find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient. [5]

Total: 14

