Solomon Practice Paper

Core Mathematics 4H

Time allowed: 90 **minutes**

Question	Points	Score
1	5	
2	6	
3	8	
4	8	
5	8	
6	10	
7	14	
8	16	
Total:	75	

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Teacher:

How I can achieve better:

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1. (a) Expand $(1+4x)^{\frac{3}{2}}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [4]

   (b) State the set of values of x for which your expansion is valid. [1]

Total: 5

2. Use the substitution $u = 1 + \sin(x)$ to find the value of [6]

$$\int_0^{\frac{\pi}{2}} \cos(x)(1+\sin(x))^3 \, \mathrm{d}x.$$

3. (a) Express [3]

$$\frac{x+11}{(x+4)(x-3)}$$

as a sum of partial fractions.

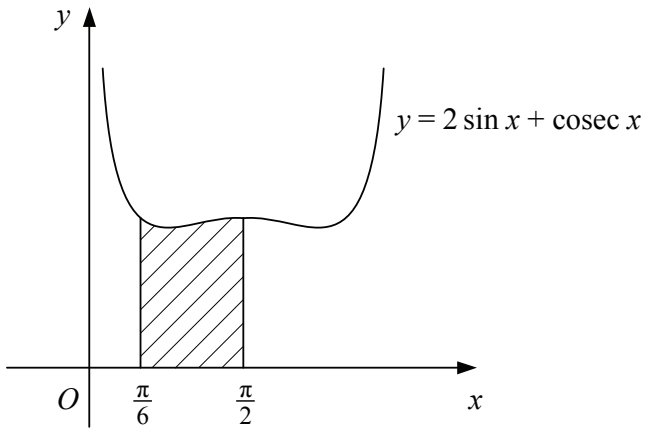
   (b) Evaluate [5]

$$\int_0^2 \frac{x+11}{(x+4)(x-3)} \, \mathrm{d}x,$$

giving your answer in the form $\ln(k)$, where k is an exact simplified fraction.

Total: 8

4. Figure shows the curve with equation $y = 2\sin(x) + \csc(x), 0 < x < \pi$. [8]



The shaded region bounded by the curve, the x-axis and the lines $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$ is rotated through 360° about the x-axis. Show that the volume of the solid formed is $\frac{1}{2}\pi(4\pi + 3\sqrt{3})$.

5. A curve has the equation

$$x^2 - 3xy - y^2 = 12.$$

   (a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

   (b) Find an equation for the tangent to the curve at the point $(2, -2)$. [3]

Total: 8

6. Relative to a fixed origin, O, the points A and B have position vectors

$$\begin{pmatrix} 1 \\ 5 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$$

respectively.

Find, in exact, simplified form,

(a) the cosine of $\angle AOB$, [4]

(b) the area of triangle OAB, [4]

(c) the shortest distance from A to the line OB. [2]

Total: 10

7. A curve has parametric equations

$$x = t(t-1), \quad \text{and} \quad y = \frac{4t}{1-t}, \qquad t \neq 1.$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [4]

The point P on the curve has parameter $t = -1$.

(b) Show that the tangent to the curve at P has the equation $x + 3y + 4 = 0$. [3]

The tangent to the curve at P meets the curve again at the point Q.

(c) Find the coordinates of Q. [7]

Total: 14

8. An entomologist is studying the population of insects in a colony.

Initially there are 300 insects in the colony and in a model, the entomologist assumes that the population, P, at time t weeks satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = kP,$$

where k is a constant.

(a) Find an expression for P in terms of k and t. [5]

Given that after one week there are 360 insects in the colony,

(b) find the value of k to 3 significant figures. [2]

Given also that after two and three weeks there are 440 and 600 insects respectively,

(c) comment on suitability of the model. [2]

An alternative model assumes that

$$\frac{\mathrm{d}P}{\mathrm{d}t} = P(0.4 - 0.25\cos(0.5t)).$$

(d) Using the initial data, $P = 300$ when $t = 0$, solve this differential equation. [4]

(e) Compare the suitability of the two models. [3]

Total: 16