Solomon Practice Paper

Core Mathematics 4G

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	7	
3	8	
4	9	
5	9	
6	10	
7	11	
8	15	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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Last updated: May 5, 2023

1. A curve has the equation [6]

$$x^2 + 2xy^2 + y = 4.$$

Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

2. Use integration by parts to find [7]

$$\int x^2 \mathrm{e}^{-x}\,\mathrm{d}x.$$

3. The first four terms in the series expansion of $(1+ax)^n$ in ascending powers of x are

$$1 - 4x + 24x^2 + kx^3,$$

where a, n and k are constants and $|ax| < 1$.

(a) Find the values of a and n. [6]

(b) Show that $k = -160$. [2]

Total: 8

4. (a) Use the trapezium rule with two intervals of equal width to find an estimate for the value of the integral [5]

$$\int_0^3 \mathrm{e}^{\cos(x)}\,\mathrm{d}x,$$

giving your answer to 3 significant figures.

(b) Use the trapezium rule with four intervals of equal width to find another estimate for the value of the integral to 3 significant figures. [2]

(c) Given that the true value of the integral lies between the estimates made in parts (*a*) and (*b*), comment on the shape of the curve $y = \mathrm{e}^{\cos(x)}$ in the interval $0 \le x \le 3$ and explain your answer. [2]

Total: 9

5. A straight road passes through villages at the points A and B with position vectors

$$(9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad (4\mathbf{j} + \mathbf{k})$$

respectively, relative to a fixed origin.

The road ends at a junction at the point C with another straight road which lies along the line with equation

$$\mathbf{r} = (2\mathbf{i} + 16\mathbf{j} - \mathbf{k}) + \mu(-5\mathbf{i} + 3\mathbf{j}),$$

where μ is a scalar parameter.

(a) Find the position vector of C. [5]

Given that 1 unit on each coordinate axis represents 200 metres,

(b) find the distance, in kilometres, from the village at A to the junction at C. [4]

Total: 9

6. A small town had a population of 9000 in the year 2001.

In a model, it is assumed that the population of the town, P, at time t years after 2001 satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.05P\mathrm{e}^{-0.05t}.$$

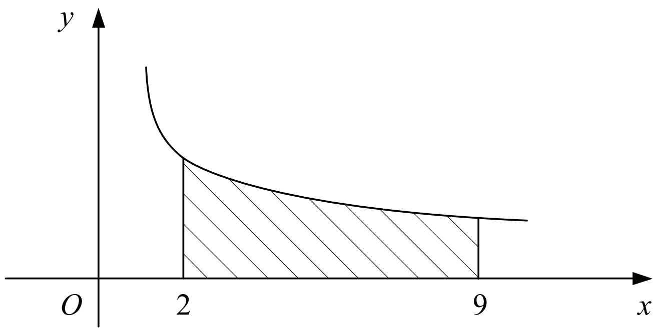
(a) Show that, according to the model, the population of the town in 2011 will be 13300 to 3 significant figures. [7]

(b) Find the value which the population of the town will approach in the long term, according to the model. [3]

Total: 10

7. Figure shows the curve with parametric equations

$$x = t^3 + 1, \quad \text{and} \quad y = \frac{2}{t}, \qquad t > 0.$$



The shaded region is bounded by the curve, the x-axis and the lines $x = 2$ and $x = 9$.

(a) Find the area of the shaded region. [5]

(b) Show that the volume of the solid formed when the shaded region is rotated through 2π radians about the x-axis is 12π. [3]

(c) Find a Cartesian equation for the curve in the form $y = \mathrm{f}(x)$. [3]

Total: 11

8. (a) Show that the substitution $u = \sin(x)$ transforms the integral [4]

$$\int \frac{6}{\cos(x)(2 - \sin(x))}\,\mathrm{d}x$$

into the integral

$$\int \frac{6}{(1 - u^2)(2 - u)}\,\mathrm{d}u.$$

(b) Express [4]

$$\frac{6}{(1-u^2)(2-u)}$$

in partial fractions.

(c) Hence, evaluate [7]

$$\int_0^{\frac{\pi}{6}} \frac{6}{\cos(x)(2-\sin(x))}\,\mathrm{d}x,$$

giving your answer in the form $a\ln(2)+b\ln(3)$, where a and b are integers.

Total: 15