Solomon Practice Paper

Core Mathematics 3K

Time allowed: 90 **minutes**

Question	Points	Score
1	8	
2	9	
3	9	
4	10	
5	12	
6	13	
7	14	
Total:	75	

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Teacher:

How I can achieve better:

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Last updated: May 5, 2023

1. (a) Find the exact value of x such that [3]

$$3\arctan(x-2)+\pi=0.$$

(b) Solve, for $-\pi<\theta<\pi$, the equation [5]

$$\cos(2\theta)-\sin(\theta)-1=0,$$

giving your answers in terms of π.

Total: 8

2. (a) Express [4]

$$\frac{4x}{x^2-9}-\frac{2}{x+3}$$

as a single fraction in its simplest form.

(b) Simplify [5]

$$\frac{x^3-8}{3x^2-8x+4}.$$

Total: 9

3. Differentiate each of the following with respect to x and simplify your answers.

(a) $\cot(x^2)$ [2]

(b) $x^2\mathrm{e}^{-x}$ [3]

(c) $\dfrac{\sin(x)}{3+2\cos(x)}$ [4]

Total: 9

4. (a) Find, as natural logarithms, the solutions of the equation [4]

$$\mathrm{e}^{2x}-8\mathrm{e}^{x}+15=0.$$

(b) Use proof by contradiction to prove that $\log_2(3)$ is irrational. [6]

Total: 10

5. The function f is defined by

$$\mathrm{f}\colon x\to 3\mathrm{e}^{x-1},\quad x\in\mathbb{R}.$$

(a) State the range of f. [1]

(b) Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. [4]

The function g is defined by

$$\mathrm{g}\colon x\to 5x-2,\quad x\in\mathbb{R}.$$

Find, in terms of e,

(c) the value of $\mathrm{gf}(\ln(2))$, [3]

(d) the solution of the equation $\mathrm{f}^{-1}\mathrm{g}(x) = 4$. [4]

Total: 12

6.

$$\mathrm{f}(x) = 2x^2 + 3\ln(2 - x) \quad x \in \mathbb{R}, x < 2.$$

(a) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form [3]

$$x = 2 - \mathrm{e}^{kx^2},$$

where k is a constant to be found.

The root, α, of the equation $\mathrm{f}(x) = 0$ is 1.9 correct to 1 decimal place.

(b) Use the iteration formula [5]

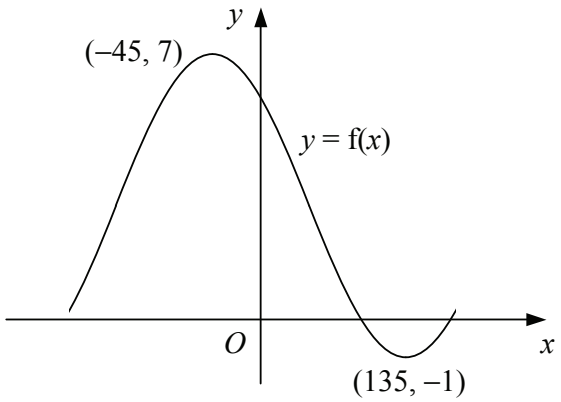
$$x_{n+1} = 2 - \mathrm{e}^{kx_n^2},$$

with $x_0 = 1.9$ and your value of k, to find α to 3 decimal places and justify the accuracy of your answer.

(c) Solve the equation $\mathrm{f}'(x) = 0$. [5]

Total: 13

7. Figure shows the curve $y = \mathrm{f}(x)$ which has



a maximum point at $(-45, 7)$ and a minimum point at $(135, -1)$.

(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of [6]

i. $y = \mathrm{f}(|x|)$,

ii. $y = 1 + 2\mathrm{f}(x)$.

Given that

$$\mathrm{f}(x) = A + 2\sqrt{2}\cos(x^\circ) - 2\sqrt{2}\sin(x^\circ), \quad x \in \mathbb{R}, -180 \leq x \leq 180,$$

where A is a constant,

(b) show that $\mathrm{f}(x)$ can be expressed in the form [3]

$$\mathrm{f}(x) = A + R\cos(x + \alpha)^\circ,$$

where $R > 0$ and $0 < \alpha < 90$,

(c) state the value of A, [1]

(d) find, to 1 decimal place, the x-coordinates of the points where the curve $y = \mathrm{f}(x)$ crosses the x-axis. [4]

Total: 14