

Solomon Practice Paper

Core Mathematics 3J

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

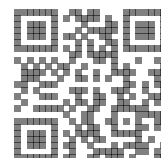
Question	Points	Score
1	8	
2	9	
3	10	
4	11	
5	11	
6	11	
7	15	
Total:	75	

How I can achieve better:

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1. (a) Given that $\cos(x) = \sqrt{3} - 1$, find the value of $\cos(2x)$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

- (b) Given that [5]

$$2 \cos(y + 30)^\circ = \sqrt{3} \sin(y - 30)^\circ,$$

find the value of $\tan(y)$ in the form $k\sqrt{3}$ where k is a rational constant.

Total: 8

2. The functions f and g are defined by

$$f(x) \equiv x^2 - 3x + 7, \quad x \in \mathbb{R},$$

$$g(x) \equiv 2x - 1, \quad x \in \mathbb{R}.$$

- (a) Find the range of f . [3]

- (b) Evaluate $gf(-1)$. [2]

- (c) Solve the equation $fg(x) = 17$. [4]

Total: 9

3.

$$f(x) = \frac{x^4 + x^3 - 13x^2 + 26x - 17}{x^2 - 3x + 3}, \quad x \in \mathbb{R}.$$

- (a) Find the values of the constants A, B, C and D such that [4]

$$f(x) = x^2 + Ax + B + \frac{Cx + D}{x^2 - 3x + 3}.$$

The point P on the curve $y = f(x)$ has x -coordinate 1.

- (b) Show that the normal to the curve $y = f(x)$ at P has the equation [6]

$$x + 5y + 9 = 0.$$

Total: 10

4. (a) Given that [5]

$$x = \sec\left(\frac{y}{2}\right), \quad 0 \leq y < \pi,$$

show that

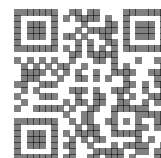
$$\frac{dy}{dx} = \frac{2}{x\sqrt{x^2 - 1}}.$$

- (b) Find an equation for the tangent to the curve $y = \sqrt{3 + 2\cos(x)}$ at the point where $x = \frac{\pi}{3}$. [6]

Total: 11

5.

$$f(x) = 5 + e^{2x-3}, \quad x \in \mathbb{R}.$$



- (a) State the range of f . [1]
- (b) Find an expression for $f^{-1}(x)$ and state its domain. [4]
- (c) Solve the equation $f(x) = 7$. [2]
- (d) Find an equation for the tangent to the curve $y = f(x)$ at the point where $y = 7$. [4]

Total: 11

6. (a) Prove the identity [5]

$$2 \cot(2x) + \tan(x) \equiv \cot(x), \quad x \neq \frac{n}{2}\pi, \quad n \in \mathbb{Z}.$$

- (b) Solve, for $0 \leq x < \pi$, the equation [6]

$$2 \cot(2x) + \tan(x) = \csc^2(x) - 7,$$

giving your answers to 2 decimal places.

Total: 11

7. The functions f and g are defined by

$$\begin{aligned} f: x &\rightarrow |2x - 5|, & x \in \mathbb{R}, \\ g: x &\rightarrow \ln(x + 3), & x \in \mathbb{R}, x > -3. \end{aligned}$$

- (a) State the range of f . [1]
- (b) Evaluate $fg(-2)$. [2]
- (c) Solve the equation $fg(x) = 3$, giving your answers in exact form. [5]
- (d) Show that the equation $f(x) = g(x)$ has a root, α , in the interval $[3, 4]$. [2]
- (e) Use the iteration formula [3]

$$x_{n+1} = \frac{1}{2} [5 + \ln(x_n + 3)],$$

with $x_0 = 3$, to find x_1, x_2, x_3 and x_4 , giving your answers to 4 significant figures.

- (f) Show that your answer for x_4 is the value of α correct to 4 significant figures. [2]

Total: 15

