Solomon Practice Paper

Core Mathematics 3C

Time allowed: 90 **minutes**

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	7	
3	8	
4	8	
5	9	
6	10	
7	13	
8	14	
Total:	75	

How I can achieve better:

-
-
-

Last updated: May 5, 2023

1. (a) Express [3]

$$\frac{x+4}{2x^2+3x+1} - \frac{2}{2x+1}$$

as a single fraction in its simplest form.

(b) Hence, find the values of x such that [3]

$$\frac{x+4}{2x^2+3x+1} - \frac{2}{2x+1} = \frac{1}{2}.$$

Total: 6

2. (a) Prove, by counter-example, that the statement [2]

$$\csc(\theta) - \sin(\theta) > 0 \text{ for all values of } \theta \text{ in the interval } 0 < \theta < \pi$$

is false.

(b) Find the values of θ in the interval $0 < \theta < \pi$ such that [5]

$$\csc(\theta) - \sin(\theta) = 2,$$

giving your answers to 2 decimal places.

Total: 7

3. Solve each equation, giving your answers in exact form.

(a) $\ln(2x-3) = 1$ [3]

(b) $3e^y + 5e^{-y} = 16$ [5]

Total: 8

4. Differentiate each of the following with respect to x and simplify your answers.

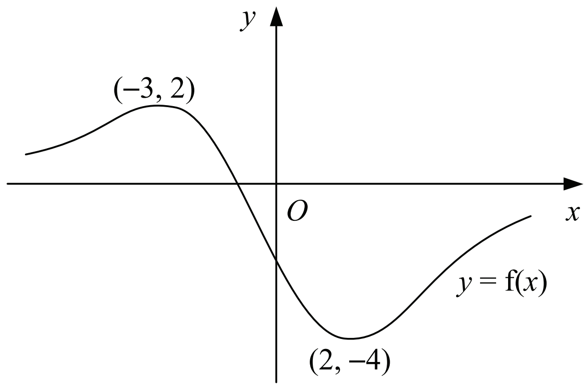
(a) $\ln(3x-2)$ [2]

(b) $\frac{2x+1}{1-x}$ [3]

(c) $x^{\frac{3}{2}}e^{2x}$ [3]

Total: 8

5. Figure shows the curve $y = f(x)$ which has a maximum point at $(-3, 2)$ and a minimum point at $(2, -4)$.



(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of [7]

i. $y = \mathrm{f}(|x|)$,

ii. $y = 3\mathrm{f}(2x)$.

(b) Write down the values of the constants a and b such that the curve with equation $y = a + \mathrm{f}(x + b)$ has a minimum point at the origin O. [2]

Total: 9

6. The function f is defined by

$$\mathrm{f}(x) \equiv 4 - \ln(3x), x \in \mathbb{R}, x > 0.$$

(a) Solve the equation $\mathrm{f}(x) = 0$. [2]

(b) Sketch the curve $y = \mathrm{f}(x)$. [2]

(c) Find an expression for the inverse function, $\mathrm{f}^{-1}(x)$. [3]

The function g is defined by

$$\mathrm{g}(x) \equiv \mathrm{e}^{2-x}, x \in \mathbb{R}.$$

(d) Show that [3]

$$\mathrm{fg}(x) = x + a - \ln(b),$$

where a and b are integers to be found.

Total: 10

7. (a) Express $4\sin(x) + 3\cos(x)$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. [4]

(b) State the minimum value of $4\sin(x) + 3\cos(x)$ and the smallest positive value of x for which this minimum value occurs. [3]

(c) Solve the equation [6]

$$4\sin(2\theta) + 3\cos(2\theta) = 2,$$

for θ in the interval $0 \le \theta \le \pi$, giving your answers to 2 decimal places.

Total: 13

8. The curve C has the equation $y = \sqrt{x} + e^{1-4x}, x \geq 0$.

   (a) Find an equation for the normal to the curve at the point $(\frac{1}{4}, \frac{3}{2})$. [4]

   The curve C has a stationary point with x-coordinate α where $0.5 < \alpha < 1$.

   (b) Show that α is a solution of the equation [3]

$$x = \frac{1}{4}\left[1 + \ln\left(8\sqrt{x}\right)\right].$$

   (c) Use the iteration formula [3]

$$x_{n+1} = \frac{1}{4}\left[1 + \ln\left(8\sqrt{x_n}\right)\right],$$

   with $x_0 = 1$ to find x_1, x_2, x_3 and x_4, giving the value of x_4 to 3 decimal places.

   (d) Show that your value for x_4 is the value of α correct to 3 decimal places. [2]

   (e) Another attempt to find α is made using the iteration formula [2]

$$x_{n+1} = \frac{1}{64}e^{8x_n - 2},$$

   with $x_0 = 1$. Describe the outcome of this attempt.

Total: 14