Solomon Practice Paper

Core Mathematics 3B

Time allowed: 90 **minutes**

Question	Points	Score
1	7	
2	9	
3	9	
4	10	
5	13	
6	13	
7	14	
Total:	75	

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1. (a) Simplify [3]

$$\frac{x^2+7x+12}{2x^2+9x+4}.$$

(b) Solve the equation [4]

$$\ln\left(x^2+7x+12\right)-1=\ln\left(2x^2+9x+4\right),$$

giving your answer in terms of e.

Total: 7

2. A curve has the equation $y=\sqrt{3x+11}$.

The point P on the curve has x-coordinate 3.

(a) Show that the tangent to the curve at P has the equation [6]

$$3x-4\sqrt{5}y+31=0.$$

The normal to the curve at P crosses the y-axis at Q.

(b) Find the y-coordinate of Q in the form $k\sqrt{5}$. [3]

Total: 9

3. (a) Use the identities for $\sin(A+B)$ and $\sin(A-B)$ to prove that [4]

$$\sin(P)+\sin(Q)\equiv 2\sin\left(\frac{P+Q}{2}\right)\cos\left(\frac{P-Q}{2}\right).$$

(b) Find, in terms of π, the solutions of the equation [5]

$$\sin(5x)+\sin(x)=0,$$

for x in the interval $0\leq x<\pi$.

Total: 9

4. The curve with equation

$$y=x^{\frac{5}{2}}\ \ln\left(\frac{x}{4}\right), x>0$$

crosses the x-axis at the point P.

(a) Write down the coordinates of P. [1]

The normal to the curve at P crosses the y-axis at the point Q.

(b) Find the area of triangle OPQ where O is the origin. [6]

The curve has a stationary point at R.

(c) Find the x-coordinate of R in exact form. [3]

Total: 10

5.

$$\mathrm{f}(x) \equiv 2x^2 + 4x + 2, \quad x \in \mathbb{R}, x \geq -1.$$

(a) Express $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$. [2]

(b) Describe fully two transformations that would map the graph of $y = x^2, x \geq 0$ onto the graph of $y = \mathrm{f}(x)$. [3]

(c) Find an expression for $\mathrm{f}^{-1}(x)$ and state its domain. [4]

(d) Sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ on the same diagram and state the relationship between them. [4]

Total: 13

6.

$$\mathrm{f}(x) = \mathrm{e}^{3x+1} - 2, \quad x \in \mathbb{R}.$$

(a) State the range of f. [1]

The curve $y = \mathrm{f}(x)$ meets the y-axis at the point P and the x-axis at the point Q.

(b) Find the exact coordinates of P and Q. [4]

(c) Show that the tangent to the curve at P has the equation [4]

$$y = 3\mathrm{e}x + \mathrm{e} - 2.$$

(d) Find to 3 significant figures the x-coordinate of the point where the tangent to the curve at P meets the tangent to the curve at Q. [4]

Total: 13

7. (a) Solve the equation [2]

$$\pi - 3\arccos(\theta) = 0.$$

(b) Sketch on the same diagram the curves [5]

$$y = \arccos(x-1), 0 \leq x \leq 2, \quad \text{and} \quad y = \sqrt{x+2}, x \geq -2.$$

Given that α is the root of the equation

$$\arccos(x-1) = \sqrt{x+2},$$

(c) show that $0 < \alpha < 1$, [3]

(d) use the iterative formula [4]

$$x_{n+1} = 1 + \cos\left(\sqrt{x_n + 2}\right)$$

with $x_0 = 1$ to find α correct to 3 decimal places.

Total: 14