Solomon Practice Paper

Core Mathematics 3A

Time allowed: 90 **minutes**

Question	Points	Score
1	4	
2	6	
3	8	
4	10	
5	10	
6	12	
7	12	
8	13	
Total:	75	

Centre: www.CasperYC.club

Name:

Teacher:

How I can achieve better:

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Last updated: May 5, 2023
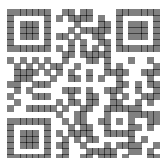
1. Given that [4]
$$x = \sec^2(y) + \tan(y),$$

   show that
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\cos^2(y)}{2\tan(y) + 1}.$$

2. The functions f and g are defined by
$$\begin{aligned} \mathrm{f} &: x \rightarrow 3x - 4, & x \in \mathbb{R}, \\ \mathrm{g} &: x \rightarrow \tfrac{2}{x+3}, & x \in \mathbb{R}, x \neq -3. \end{aligned}$$

   (a) Evaluate fg(1). [2]

   (b) Solve the equation gf$(x) = 6$. [4]

   Total: 6

3. Giving your answers to 2 decimal places, solve the simultaneous equations [8]
$$\begin{aligned} \mathrm{e}^{2y} - x + 2 &= 0 \\ \ln(x + 3) - 2y - 1 &= 0 \end{aligned}$$

4. (a) Use the derivatives of $\sin(x)$ and $\cos(x)$ to prove that [4]
$$\frac{\mathrm{d}}{\mathrm{d}x}\tan(x) = \sec^2(x).$$

   The tangent to the curve $y = 2x\tan(x)$ at the point where $x = \frac{\pi}{4}$ meets the y-axis at the point P.

   (b) Find the y-coordinate of P in the form $k\pi^2$ where k is a rational constant. [6]

   Total: 10

5. (a) Express [4]
$$3\cos(x^\circ) + \sin(x^\circ)$$

   in the form $R\cos(x - \alpha)^\circ$ where $R > 0$ and $0 < \alpha < 90$.

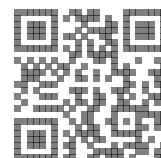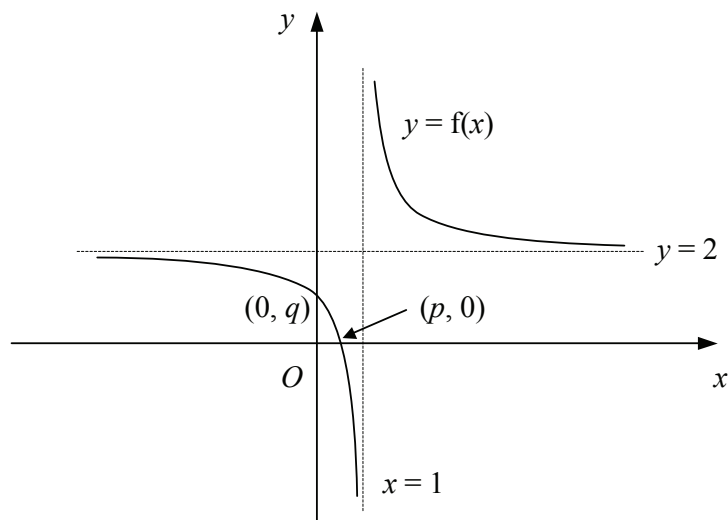   (b) Using your answer to part (a), or otherwise, solve the equation [6]
$$6\cos^2(x^\circ) + \sin(2x^\circ) = 0,$$

   for x in the interval $0 \leq x \leq 360$, giving your answers to 1 decimal place where appropriate.

   Total: 10

6. Figure shows the curve with equation $y = \mathrm{f}(x)$.

The curve crosses the axes at $(p, 0)$ and $(0, q)$ and the lines $x = 1$ and $y = 2$ are asymptotes of the curve.

(a) Showing the coordinates of any points of intersection with the axes and the equations of any asymptotes, sketch on separate diagrams the graphs of [6]

    i. $y = |\mathrm{f}(x)|$,

    ii. $y = 2\mathrm{f}(x + 1)$.

Given also that

$$\mathrm{f}(x) \equiv \frac{2x - 1}{x - 1}, \quad x \in \mathbb{R}, x \neq 1,$$

(b) find the values of p and q, [3]

(c) find an expression for $\mathrm{f}^{-1}(x)$. [3]

Total: 12

7. (a)   i. Show that [6]

$$\sin(x + 30)° + \sin(x - 30)° \equiv a \sin(x°),$$

    where a is a constant to be found.

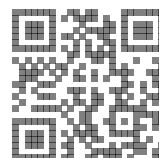   ii. Hence find the exact value of $\sin(75°) + \sin(15°)$, giving your answer in the form $b\sqrt{6}$.

(b) Solve, for $0 \leq y \leq 360$, the equation [6]

$$2\cot^2(y°) + 5\csc(y°) + \csc^2(y°) = 0.$$

Total: 12

8.

$$\mathrm{f}(x) = \frac{x^4 + x^3 - 5x^2 - 9}{x^2 + x - 6}.$$

(a) Using algebraic division, show that [5]

$$f(x) = x^2 + A + \frac{B}{x + C},$$

where A, B and C are integers to be found.

(b) By sketching two suitable graphs on the same set of axes, show that the equation $f(x) = 0$ has exactly one real root. [3]

(c) Use the iterative formula [5]

$$x_{n+1} = 2 + \frac{1}{x_n^2 + 1},$$

with a suitable starting value to find the root of the equation $f(x) = 0$ correct to 3 significant figures and justify the accuracy of your answer.

Total: 13