Solomon Practice Paper

Core Mathematics 4L

Time allowed: 90 **minutes**

Question	Points	Score
1	8	
2	8	
3	10	
4	11	
5	12	
6	12	
7	14	
Total:	75	

Centre: **www.CasperYC.club**

Name:

Teacher:

How I can achieve better:

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Last updated: May 5, 2023

1. The number of people, n, in a queue at a Post Office t minutes after it opens is modelled by the differential equation

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \mathrm{e}^{0.5t} - 5, \qquad t \geq 0.$$

   (a) Find, to the nearest second, the time when the model predicts that there will be the least number of people in the queue. [3]

   (b) Given that there are 20 people in the queue when the Post Office opens, solve the differential equation. [4]

   (c) Explain why this model would not be appropriate for large values of t. [1]

Total: 8

2. A curve has the equation [8]

$$3x^2 + xy - 2y^2 + 25 = 0.$$

Find an equation for the normal to the curve at the point with coordinates $(1, 4)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

3. (a) Use the substitution $u = 2 - x^2$ to find [4]

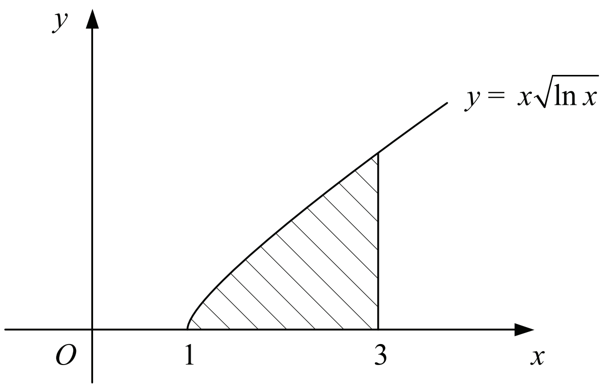
$$\int \frac{x}{2 - x^2} \, \mathrm{d}x.$$

(b) Evaluate [6]

$$\int_0^{\frac{\pi}{4}} \sin(3x)\cos(x) \, \mathrm{d}x.$$

Total: 10

4. Figure shows the curve with equation $y = x\sqrt{\ln(x)}, x \geq 1$.



The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(a) Using the trapezium rule with two intervals of equal width, estimate the area of the shaded region. [4]

The shaded region is rotated through 360° about the x-axis.

(b) Find the exact volume of the solid formed. [7]

Total: 11

5.

$$f(x) = \frac{5 - 8x}{(1 + 2x)(1 - x)^2}.$$

(a) Express $f(x)$ in partial fractions. [5]

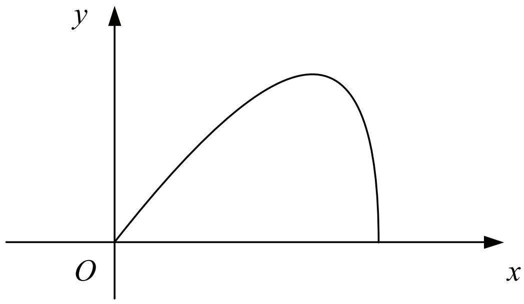
(b) Find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [6]

(c) State the set of values of x for which your expansion is valid. [1]

Total: 12

6. Figure shows the curve with parametric equations

$$x = t + \sin(t), \quad \text{and} \quad y = \sin(t), \qquad 0 \le t \le \pi.$$



(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [3]

(b) Find, in exact form, the coordinates of the point where the tangent to the curve is parallel to the x-axis. [3]

(c) Show that the region bounded by the curve and the x-axis has area 2. [6]

Total: 12

7. The line l_1 passes through the points A and B with position vectors $(3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k})$ and $(8\mathbf{j} - 6\mathbf{k})$ respectively, relative to a fixed origin.

   (a) Find a vector equation for l_1. [2]

   The line l_2 has vector equation

$$\mathbf{r} = (-2\mathbf{i} + 10\mathbf{j} + 6\mathbf{k}) + \mu(7\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}),$$

   where μ is a scalar parameter.

   (b) Show that lines l_1 and l_2 intersect. [4]

   (c) Find the coordinates of the point where l_1 and l_2 intersect. [2]

   The point C lies on l_2 and is such that AC is perpendicular to AB.

   (d) Find the position vector of C. [6]

Total: 14