Solomon Practice Paper

Core Mathematics 4J

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	6	
3	8	
4	9	
5	9	
6	10	
7	13	
8	14	
Total:	75	

Centre: www.CasperYC.club

Name:

Teacher:

How I can achieve better:

-
-
-

Last updated: May 5, 2023

1. The region bounded by the curve $y = x^2 - 2x$ and the x-axis is rotated through 2π radians about the x-axis. [6]

   Find the volume of the solid formed, giving your answer in terms of π.

2. Use the substitution $u = 1 - x^{\frac{1}{2}}$ to find [6]

$$\int \frac{1}{1 - x^{\frac{1}{2}}} \, \mathrm{d}x.$$

3. A curve has the equation

$$2\sin(2x) - \tan(y) = 0.$$

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form in terms of x and y. [5]

(b) Show that the tangent to the curve at the point $\left(\frac{\pi}{6}, \frac{\pi}{3}\right)$ has the equation [3]

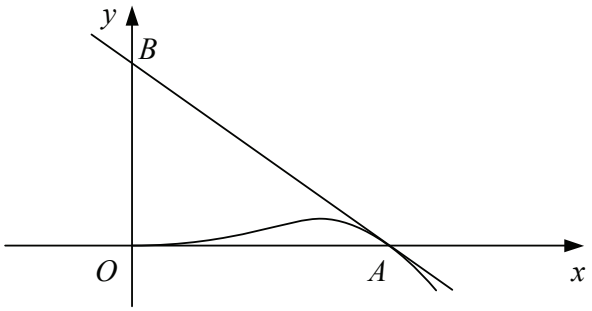
$$y = \frac{1}{2}x + \frac{\pi}{4}.$$

Total: 8

4. Figure shows the curve with parametric equations

$$x = a\sqrt{t}, \quad \text{and} \quad y = at(1-t), \qquad t \geq 0,$$

where a is a positive constant.



(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t. [3]

The curve meets the x-axis at the origin, O, and at the point A. The tangent to the curve at A meets the y-axis at the point B as shown.

(b) Show that the area of triangle OAB is a^2. [6]

Total: 9

5. The gradient at any point (x, y) on a curve is proportional to $\sqrt{y}$.

   Given that the curve passes through the point with coordinates $(0, 4)$,

   (a) show that the equation of the curve can be written in the form [5]

   $$2\sqrt{y} = kx + 4,$$

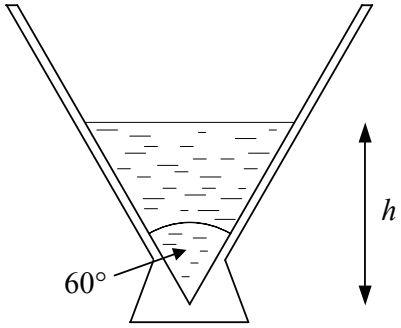
   where k is a positive constant.

   Given also that the curve passes through the point with coordinates $(2, 9)$,

   (b) find the equation of the curve in the form $y = \mathrm{f}(x)$. [4]

Total: 9

6. Figure shows a vertical cross-section of a vase.



The inside of the vase is in the shape of a right-circular cone with the angle between the sides in the cross-section being 60°. When the depth of water in the vase is h cm, the volume of water in the vase is V cm^3.

(a) Show that $V = \frac{1}{9}\pi h^3$. [3]

The vase is initially empty and water is poured in at a constant rate of 120 cm^3 s^{-1}.

(b) Find, to 2 decimal places, the rate at which h is increasing [7]

i. when $h = 6$,

ii. after water has been poured in for 8 seconds.

Total: 10

7. Relative to a fixed origin, the points A and B have position vectors

$$\begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -3 \\ 6 \\ 1 \end{pmatrix}$$

respectively.

(a) Find a vector equation for the line l_1 which passes through A and B. [2]

The line l_2 has vector equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -7 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

(b) Show that lines l_1 and l_2 do not intersect. [5]

(c) Find the position vector of the point C on l_2 such that $\angle ABC = 90°$. [6]

Total: 13

8.
$$\mathrm{f}(x) = \frac{x(3x-7)}{(1-x)(1-3x)}, \qquad |x| < \frac{1}{3}.$$

(a) Find the values of the constants A, B and C such that [4]
$$\mathrm{f}(x) = A + \frac{B}{1-x} + \frac{C}{1-3x}.$$

(b) Evaluate [5]
$$\int_0^{\frac{1}{4}} \mathrm{f}(x)\,\mathrm{d}x,$$
giving your answer in the form $p + \ln(q)$, where p and q are rational.

(c) Find the series expansion of $\mathrm{f}(x)$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. [5]

Total: 14