

Solomon Practice Paper

Core Mathematics 4E

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	4	
2	8	
3	9	
4	9	
5	10	
6	10	
7	12	
8	13	
Total:	75	

How I can achieve better:

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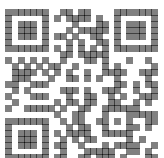
Last updated: May 5, 2023



1. Find

$$\int \cot^2(2x) \, dx.$$

[4]



2. A curve has the equation

$$4 \cos(x) + 2 \sin(y) = 3.$$

(a) Show that

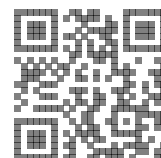
$$\frac{dy}{dx} = 2 \sin(x) \sec(y).$$

[5]

(b) Find an equation for the tangent to the curve at the point $(\frac{\pi}{3}, \frac{\pi}{6})$, giving your answer in the form $ax + by = c$, where a and b are integers.

[3]

Total: 8



3. (a) Express

$$\frac{2 + 20x}{1 + 2x - 8x^2}$$

[4]

as a sum of partial fractions.

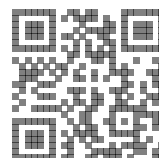
(b) Hence find the series expansion of

[5]

$$\frac{2 + 20x}{1 + 2x - 8x^2}, \quad |x| < \frac{1}{4},$$

in ascending powers of x up to and including the term in x^3 , simplifying each coefficient.

Total: 9



4. The line l_1 passes through the points P and Q with position vectors $(-\mathbf{i}-8\mathbf{j}+3\mathbf{k})$ and $(2\mathbf{i}-9\mathbf{j}+\mathbf{k})$ respectively, relative to a fixed origin.

(a) Find a vector equation for l_1 .

[2]

The line l_2 has the equation

$$\mathbf{r} = (6\mathbf{i} + a\mathbf{j} + b\mathbf{k}) + \mu(\mathbf{i} + 4\mathbf{j} - \mathbf{k})$$

and also passes through the point Q .

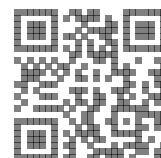
(b) Find the values of the constants a and b .

[3]

(c) Find, in degrees to 1 decimal place, the acute angle between lines l_1 and l_2 .

[4]

Total: 9



5. At time $t = 0$, a tank of height 2 metres is completely filled with water. Water then leaks from a hole in the side of the tank such that the depth of water in the tank, y metres, after t hours satisfies the differential equation

$$\frac{dy}{dt} = -ke^{-0.2t},$$

where k is a positive constant,

- (a) Find an expression for y in terms of k and t . [4]

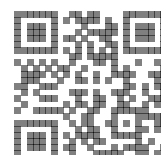
Given that two hours after being filled the depth of water in the tank is 1.6 metres,

- (b) find the value of k to 4 significant figures. [3]

Given also that the hole in the tank is h cm above the base of the tank,

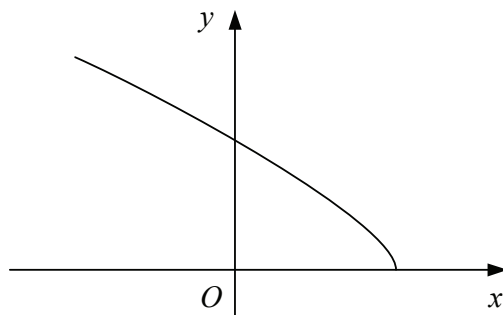
- (c) show that $h = 79$ to 2 significant figures. [3]

Total: 10



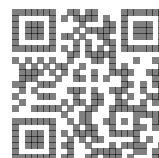
6. Figure shows the curve with parametric equations

$$x = 2 - t^2, \quad \text{and} \quad y = t(t + 1), \quad t \geq 0.$$



- (a) Find the coordinates of the points where the curve meets the coordinate axes. [4]
- (b) Find the exact area of the region bounded by the curve and the coordinate axes. [6]

Total: 10



7. (a) Prove that

$$\frac{d}{dx}a^x = a^x \ln(a).$$

[3]

A curve has the equation $4^x - 2^{x-1} + 1$.

(b) Show that the tangent to the curve at the point where it crosses the y -axis has the equation

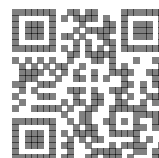
[5]

$$3x \ln(2) - 2y + 3 = 0.$$

(c) Find the exact coordinates of the stationary point of the curve.

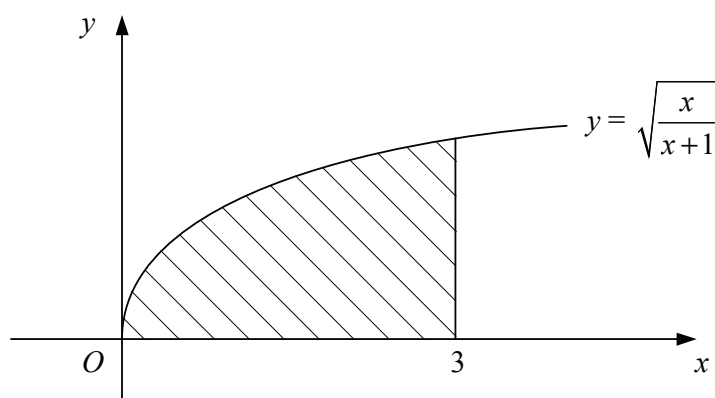
[4]

Total: 12



8. Figure shows the curve with equation

$$y = \sqrt{\frac{x}{x+1}}.$$



The shaded region is bounded by the curve, the x -axis and the line $x = 3$.

- (a) i. Use the trapezium rule with three strips to find an estimate for the area of the shaded region. [7]
- ii. Use the trapezium rule with six strips to find an improved estimate for the area of the shaded region.

The shaded region is rotated through 2π radians about the x -axis.

- (b) Show that the volume of the solid formed is $\pi(3 - \ln(4))$. [6]

Total: 13

