Solomon Practice Paper

Core Mathematics 4D

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

Question	Points	Score
1	6	
2	8	
3	10	
4	12	
5	12	
6	13	
7	14	
Total:	75	

How I can achieve better:

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((a) Find the binomial expansion of $(2-3x)^{-3}$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient.	9		
((b) State the set of values of x for which your expansion is valid.			
		Tot		

2. A curve has the equation

$$x^2 + 3xy - 2y^2 + 17 = 0.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [5] (b) Find an equation for the normal to the curve at the point (3, -2). [3]

Core Mathematics – Practice Paper 4D

3. (a) Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \equiv Ax + B + \frac{C}{x} + \frac{D}{x - 3}.$$

(b) Evaluate

$$\int_{1}^{2} \frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \,\mathrm{d}x,$$

giving your answer in the form $p + q \ln(2)$, where p and q are integers.

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[5]

[5]

4. A mathematician is selling goods at a car boot sale. She believes that the rate at which she makes sales depends on the length of time since the start of the sale, t hours, and the total value of sales she has made up to that time, $\pounds x$.

She uses the model

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k(5-t)}{x},$$

where k is a constant.

Given that after two hours she has made sales of $\pounds 96$ in total,

(a) solve the differential equation and show that she made $\pounds 72$ in the first hour of the sale. [8]

The mathematician believes that is it not worth staying at the sale once she is making sales at a rate of less than $\pounds 10$ per hour.

(b) Verify that at 3 hours and 5 minutes after the start of the sale, she should have already left. [4]

5. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 4\\1\\1 \end{pmatrix} + s \begin{pmatrix} 1\\4\\5 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} -3\\1\\-6 \end{pmatrix} + t \begin{pmatrix} 3\\a\\b \end{pmatrix},$$

where a and b are constants and s and t are scalar parameters.

Given that the two lines are perpendicular,

(a) find a linear relationship between a and b.

Given also that the two lines intersect,

- (b) find the values of a and b,
- (c) find the coordinates of the point where they intersect.

Total: 12

[2]

[8]

[2]

6. Figure shows the curve with equation



- (a) Use the substitution $u^2 = 1 x$ to show that the area of the region bounded by the curve [8] and the x-axis is $\frac{4}{15}$.
- (b) Find, in terms of π , the volume of the solid formed when the region bounded by the curve [5] and the x-axis is rotated through 360° about the x-axis.



7. A curve has parametric equations

$$x = 3\cos^2(t)$$
, and $y = \sin(2t)$, $0 \le t < \pi$.

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{3}\cot(2t).$$
[4]

(b) Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. [3]

(c) Show that the tangent to the curve at the point where $t = \frac{\pi}{6}$ has the equation [3]

$$2x + 3\sqrt{3}y = 9.$$

(d) Find a Cartesian equation for the curve in the form $y^2 = f(x)$.

Total: 14

[4]