

Solomon Practice Paper

Core Mathematics 4C

Time allowed: 90 minutes

Centre: www.CasperYC.club

Name:

Teacher:

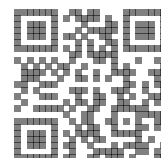
Question	Points	Score
1	6	
2	7	
3	11	
4	11	
5	13	
6	13	
7	14	
Total:	75	

How I can achieve better:

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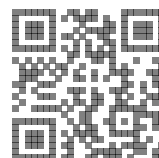
Last updated: May 5, 2023



1. Use integration by parts to show that

[6]

$$\int_1^2 x \ln(x) \, dx = 2 \ln(2) - \frac{3}{4}.$$

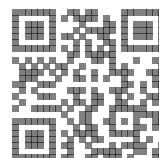


2. (a) Use the trapezium rule with two intervals of equal width to find an approximate value for the integral [5]

$$\int_0^2 \arctan(x) dx.$$

- (b) Use the trapezium rule with four intervals of equal width to find an improved approximation for the value of the integral. [2]

Total: 7



3. A curve has the equation

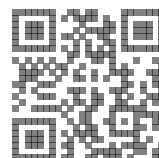
$$3x^2 - 2x + xy + y^2 - 11 = 0.$$

The point P on the curve has coordinates $(-1, 3)$.

(a) Show that the normal to the curve at P has the equation $y = 2 - x$. [7]

(b) Find the coordinates of the point where the normal to the curve at P meets the curve again. [4]

Total: 11



4. The points A and B have coordinates $(3, 9, -7)$ and $(13, -6, -2)$ respectively.

(a) Find, in vector form, an equation for the line l which passes through A and B . [2]

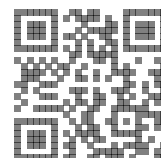
(b) Show that the point C with coordinates $(9, 0, -4)$ lies on l . [2]

The point D is the point on l closest to the origin, O .

(c) Find the coordinates of D . [4]

(d) Find the area of triangle OAB to 3 significant figures. [3]

Total: 11



5. A bath is filled with hot water which is allowed to cool. The temperature of the water is $\theta^\circ\text{C}$ after cooling for t minutes and the temperature of the room is assumed to remain constant at 20°C .

Given that the rate at which the temperature of the water decreases is proportional to the difference in temperature between the water and the room,

- (a) write down a differential equation connecting θ and t . [2]

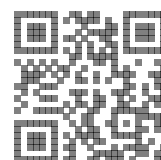
Given also that the temperature of the water is initially 37°C and that it is 36°C after cooling for four minutes,

- (b) find, to 3 significant figures, the temperature of the water after ten minutes. [8]

Advice suggests that the temperature of the water should be allowed to cool to 33°C before a child gets in.

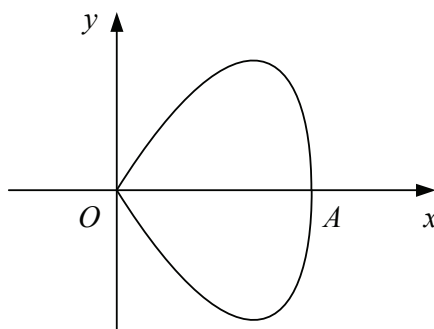
- (c) Find, to the nearest second, how long a child should wait before getting into the bath. [3]

Total: 13



6. Figure shows the curve with parametric equations

$$x = 3 \sin(t) \quad \text{and} \quad y = 2 \sin(2t), \quad 0 \leq t < \pi.$$



The curve meets the x -axis at the origin, O , and at the point A .

(a) Find the value of t at O and the value of t at A . [2]

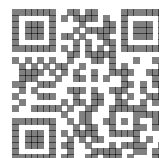
The region enclosed by the curve is rotated through π radians about the x -axis.

(b) Show that the volume of the solid formed is given by [3]

$$\int_0^{\frac{\pi}{2}} 12\pi \sin^2(2t) \cos(t) dt.$$

(c) Using the substitution $u = \sin(t)$, or otherwise, evaluate this integral, giving your answer as an exact multiple of π . [8]

Total: 13



7.

$$f(x) = \frac{8 - x}{(1 + x)(2 - x)}, \quad |x| < 1.$$

(a) Express $f(x)$ in partial fractions. [3]

(b) Show that [5]

$$\int_0^{\frac{1}{2}} f(x) \, dx = \ln(k),$$

where k is an integer to be found.

(c) Find the series expansion of $f(x)$ in ascending powers of x up to and including the term in x^3 , simplifying each coefficient. [6]

Total: 14

