Solomon Practice Paper

Core Mathematics 3L

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	7	
3	8	
4	9	
5	9	
6	10	
7	12	
8	14	
Total:	75	

Centre: www.CasperYC.club

Name:

Teacher:

How I can achieve better:

-
-
-

Last updated: May 5, 2023

1.

$$\mathrm{f}(x) \equiv \frac{2x-3}{x-2}, \quad x \in \mathbb{R}, x > 2.$$

(a) Find the range of f. [2]

(b) Show that $\mathrm{ff}(x) = x$ for all $x > 2$. [3]

(c) Hence, write down an expression for $\mathrm{f}^{-1}(x)$. [1]

Total: 6

2. Solve each equation, giving your answers in exact form.

   (a) $e^{4x-3} = 2$ [3]

   (b) $\ln(2y-1) = 1 + \ln(3-y)$ [4]

Total: 7

3. The curve C has the equation $y = 2e^x - 6\ln(x)$ and passes through the point P with x-coordinate 1.

   (a) Find an equation for the tangent to C at P. [4]

   The tangent to C at P meets the coordinate axes at the points Q and R.

   (b) Show that the area of triangle OQR, where O is the origin, is $\dfrac{9}{3-e}$. [4]

Total: 8

4. (a) Express [5]

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)}$$

as a single fraction in its simplest form.

(b) Hence, show that the equation [4]

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)} = 1$$

has no real roots.

Total: 9

5. Find the values of x in the interval $-180° < x < 180°$ for which [9]

$$\tan(x + 45)° - \tan(x)° = 4,$$

giving your answers to 1 decimal place.

6. (a) Sketch on the same diagram the graphs of [6]

$$y = |x| - a \quad \text{and} \quad y = |3x + 5a|,$$

where a is a positive constant.

Show on your diagram the coordinates of any points where each graph meets the coordinate axes.

(b) Solve the equation [4]

$$|x| - a = |3x + 5a|.$$

Total: 10

7. (a) Use the identity [3]

$$\cos(A+B) \equiv \cos(A)\cos(B) - \sin(A)\sin(B)$$

to prove that

$$\cos(x) \equiv 1 - 2\sin^2\left(\frac{x}{2}\right).$$

(b) Prove that, for $\sin(x) \neq 0$, [3]

$$\frac{1-\cos(x)}{\sin(x)} \equiv \tan\left(\frac{x}{2}\right).$$

(c) Find the values of x in the interval $0 \leq x \leq 360°$ for which [6]

$$\frac{1-\cos(x)}{\sin(x)} = 2\sec^2\left(\frac{x}{2}\right) - 5.$$

giving your answers to 1 decimal place where appropriate.

Total: 12

8. A curve has the equation $y = (2x + 3)\mathrm{e}^{-x}$.

   (a) Find the exact coordinates of the stationary point of the curve. [4]

   The curve crosses the y-axis at the point P.

   (b) Find an equation for the normal to the curve at P. [2]

   The normal to the curve at P meets the curve again at Q.

   (c) Show that the x-coordinate of Q lies in the interval $[-2, -1]$. [3]

   (d) Use the iterative formula [3]

   $$x_{n+1} = \frac{3 - 3\mathrm{e}^{x_n}}{\mathrm{e}^{x_n} - 2},$$

   with $x_0 = -1$, to find x_1, x_2, x_3 and x_4. Give the value of x_4 to 2 decimal places.

   (e) Show that your value for x_4 is the x-coordinate of Q correct to 2 decimal places. [2]

Total: 14