Solomon Practice Paper

Core Mathematics 3J

Time allowed: 90 **minutes**

Centre: **www.CasperYC.club**

Name:

Teacher:

Question	Points	Score
1	8	
2	9	
3	10	
4	11	
5	11	
6	11	
7	15	
Total:	75	

How I can achieve better:

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BY NC ND

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1. (a) Given that $\cos(x) = \sqrt{3} - 1$, find the value of $\cos(2x)$ in the form $a + b\sqrt{3}$, where a and b are integers. [3]

   (b) Given that [5]

$$2\cos(y + 30)^\circ = \sqrt{3}\sin(y - 30)^\circ,$$

   find the value of $\tan(y)$ in the form $k\sqrt{3}$ where k is a rational constant.

Total: 8

2. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}(x) &\equiv x^2 - 3x + 7, \quad x \in \mathbb{R}, \\ \mathrm{g}(x) &\equiv 2x - 1, \quad\quad\quad x \in \mathbb{R}. \end{aligned}$$

(a) Find the range of f. [3]

(b) Evaluate $\mathrm{gf}(-1)$. [2]

(c) Solve the equation $\mathrm{fg}(x) = 17$. [4]

Total: 9

3.
$$\mathrm{f}(x) = \frac{x^4 + x^3 - 13x^2 + 26x - 17}{x^2 - 3x + 3}, x \in \mathbb{R}.$$

(a) Find the values of the constants A, B, C and D such that [4]

$$\mathrm{f}(x) = x^2 + Ax + B + \frac{Cx + D}{x^2 - 3x + 3}.$$

The point P on the curve $y = \mathrm{f}(x)$ has x-coordinate 1.

(b) Show that the normal to the curve $y = \mathrm{f}(x)$ at P has the equation [6]

$$x + 5y + 9 = 0.$$

Total: 10

4. (a) Given that [5]

$$x = \sec\left(\frac{y}{2}\right), 0 \leq y < \pi,$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{x\sqrt{x^2-1}}.$$

(b) Find an equation for the tangent to the curve $y = \sqrt{3 + 2\cos(x)}$ at the point where $x = \frac{\pi}{3}$. [6]

Total: 11

5.

$$f(x) = 5 + e^{2x-3}, x \in \mathbb{R}.$$

(a) State the range of f. [1]

(b) Find an expression for $f^{-1}(x)$ and state its domain. [4]

(c) Solve the equation $f(x) = 7$. [2]

(d) Find an equation for the tangent to the curve $y = f(x)$ at the point where $y = 7$. [4]

Total: 11

6. (a) Prove the identity [5]

$$2\cot(2x) + \tan(x) \equiv \cot(x), \quad x \neq \frac{n}{2}\pi, \quad n \in \mathbb{Z}.$$

(b) Solve, for $0 \le x < \pi$, the equation [6]

$$2\cot(2x) + \tan(x) = \csc^2(x) - 7,$$

giving your answers to 2 decimal places.

Total: 11

7. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}\colon x &\to |2x-5|, \quad x \in \mathbb{R}, \\ \mathrm{g}\colon x &\to \ln(x+3), \quad x \in \mathbb{R}, x > -3. \end{aligned}$$

(a) State the range of f. [1]

(b) Evaluate $\mathrm{fg}(-2)$. [2]

(c) Solve the equation $\mathrm{fg}(x) = 3$, giving your answers in exact form. [5]

(d) Show that the equation $\mathrm{f}(x) = \mathrm{g}(x)$ has a root, α, in the interval $[3, 4]$. [2]

(e) Use the iteration formula [3]

$$x_{n+1} = \frac{1}{2}\left[5 + \ln(x_n + 3)\right],$$

with $x_0 = 3$, to find x_1, x_2, x_3 and x_4, giving your answers to 4 significant figures.

(f) Show that your answer for x_4 is the value of α correct to 4 significant figures. [2]

Total: 15