Solomon Practice Paper

Core Mathematics 3H

Time allowed: 90 **minutes**

Question	Points	Score
1	6	
2	7	
3	7	
4	9	
5	10	
6	11	
7	12	
8	13	
Total:	75	

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Name:

Teacher:

How I can achieve better:

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1. The functions f and g are defined by

$$\begin{aligned} \mathrm{f}: x &\to 2 - x^2, \quad x \in \mathbb{R}, \\ \mathrm{g}: x &\to \tfrac{3x}{2x-1}, \quad x \in \mathbb{R}, x \neq \tfrac{1}{2}. \end{aligned}$$

(a) Evaluate fg(2). [2]

(b) Solve the equation $\mathrm{gf}(x) = \frac{1}{2}$. [4]

Total: 6

2. Giving your answers to 1 decimal place, solve the equation [7]

$$5\tan^2(2\theta) - 13\sec(2\theta) = 1,$$

for θ in the interval $0 \leq \theta \leq 360°$.

3. (a) Simplify [3]

$$\frac{2x^2+3x-9}{2x^2-7x+6}.$$

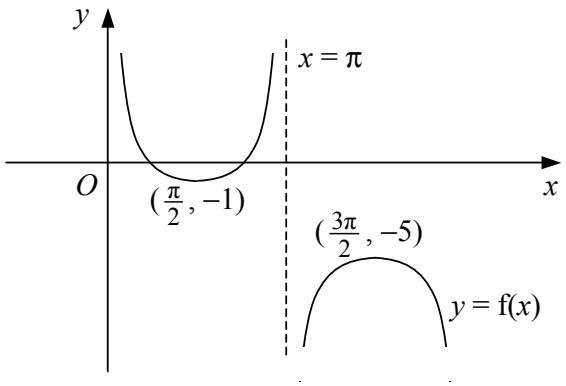
(b) Solve the equation [4]

$$\ln(2x^2+3x-9) = 2+\ln(2x^2-7x+6),$$

giving your answer in terms of e.

Total: 7

4. Figure shows the graph of $y = \mathrm{f}(x)$.



The graph has a minimum at $\left(\frac{\pi}{2}, -1\right)$, a maximum at $\left(\frac{3\pi}{2}, -5\right)$ and an asymptote with equation $x = \pi$.

(a) Showing the coordinates of any stationary points, sketch the graph of $y = |\mathrm{f}(x)|$. [3]

Given that

$$\mathrm{f}: x \to a + b\csc(x), \quad x \in \mathbb{R}, 0 < x < 2\pi, x \neq \pi,$$

(b) find the values of the constants a and b, [3]

(c) find, to 2 decimal places, the x-coordinates of the points where the graph of $y = \mathrm{f}(x)$ crosses the x-axis. [3]

Total: 9

5. The number of bacteria present in a culture at time t hours is modelled by the continuous variable N and the relationship

$$N = 2000\mathrm{e}^{kt},$$

where k is a constant.

Given that when $t = 3, N = 18000$, find

(a) the value of k to 3 significant figures, [3]

(b) how long it takes for the number of bacteria present to double, giving your answer to the nearest minute, [4]

(c) the rate at which the number of bacteria is increasing when $t = 3$. [3]

Total: 10

6. (a) Use the derivative of $\cos(x)$ to prove that [4]

$$\frac{\mathrm{d}}{\mathrm{d}x}\sec(x) = \sec(x)\tan(x).$$

The curve C has the equation

$$y = \mathrm{e}^{2x}\sec(x), -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

(b) Find an equation for the tangent to C at the point where it crosses the y-axis. [4]

(c) Find, to 2 decimal places, the x-coordinate of the stationary point of C. [3]

Total: 11

7.

$$\mathrm{f}(x) = x^2 - 2x + 5, x \in \mathbb{R}, x \geq 1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(b) State the range of f. [1]

(c) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(d) Describe fully two transformations that would map the graph of $y = \mathrm{f}^{-1}(x)$ onto the graph of $y = \sqrt{x}, x \geq 0$. [2]

(e) Find an equation for the normal to the curve $y = \mathrm{f}^{-1}(x)$ at the point where $x = 8$. [4]

Total: 12

8. A curve has the equation

$$y = \frac{e^2}{x} + e^x, x \neq 0.$$

(a) Find $\frac{dy}{dx}$. [2]

(b) Show that the curve has a stationary point in the interval $[1.3, 1.4]$. [3]

The point A on the curve has x-coordinate 2.

(c) Show that the tangent to the curve at A passes through the origin. [4]

The tangent to the curve at A intersects the curve again at the point B.

The x-coordinate of B is to be estimated using the iterative formula

$$x_{n+1} = -\frac{2}{3}\sqrt{3 + 3x_n e^{x_n - 2}},$$

with $x_0 = -1$.

(d) Find x_1, x_2 and x_3 to 7 significant figures and hence state the x-coordinate of B to 5 significant figures. [4]

Total: 13